

Peculiar properties of a dirty superconductor/low resistive normal metal hybrid

D. Yu. Vodolazov

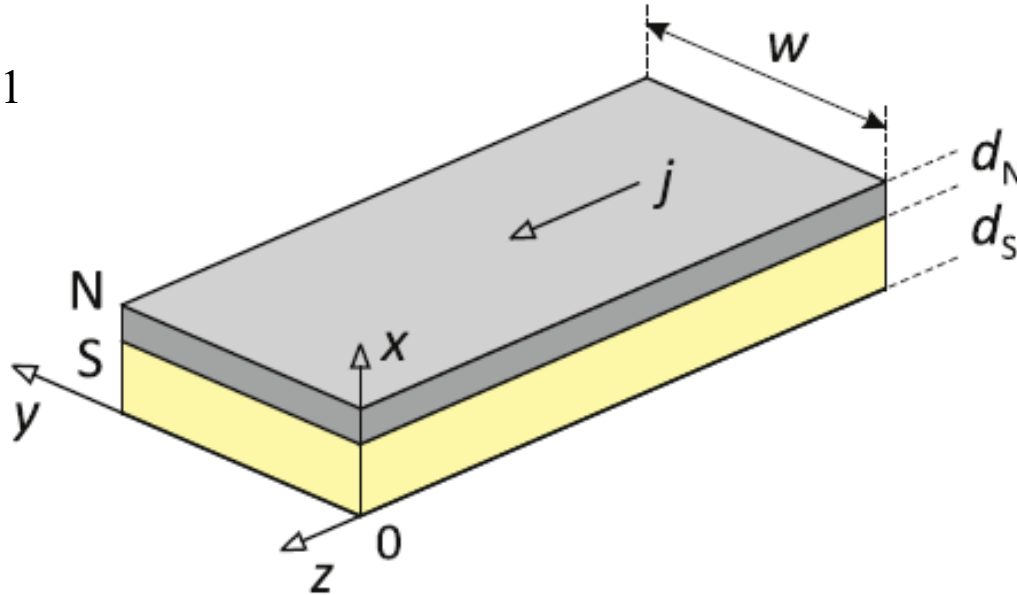
Institute for Physics of Microstructures, 603960 Nizhny Novgorod, Russian Academy of Science, Russia

Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia

Subject of the talk: unique superconducting properties of superconductor(S)/normal metal (N) hybrid

$$d_N \sim d_S \sim \xi_c, \quad \xi_c = (\hbar D_S / k_B T_{c0})^{1/2}$$

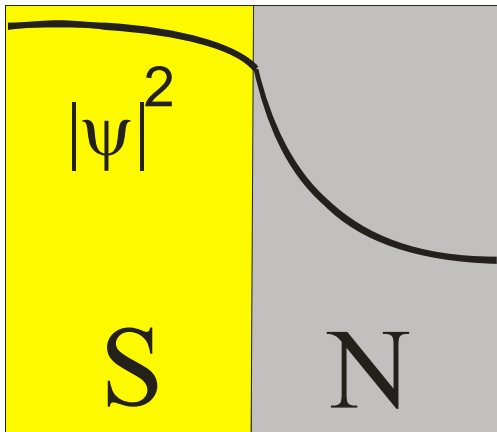
$$\rho_S \gg \rho_N, \quad D_N / D_S \gg 1$$



$$q = \nabla\phi + 2\pi A/\Phi_0 \quad v = \hbar q/m$$

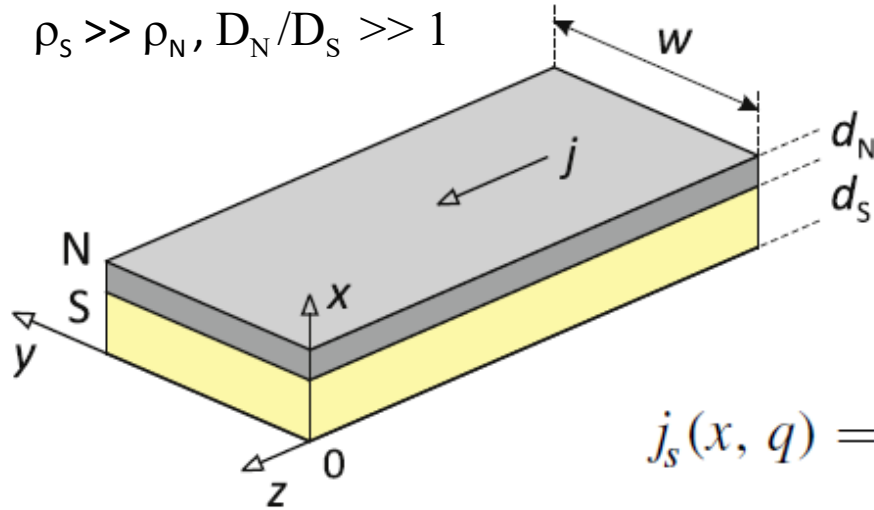
$$j_s(x, q) = -q \frac{c\Phi_0}{8\pi^2} \frac{1}{\lambda^2(x, q)}$$

$$\frac{1}{\lambda^2(x, q)} = \frac{16\pi^2 k_B T}{\hbar c^2} \times \frac{1}{\rho} \sum_{\omega_n \geq 0} \sin^2 \Theta(x, q)$$



Proximity induced
superconductivity in N layer

Screening properties of SN bilayer

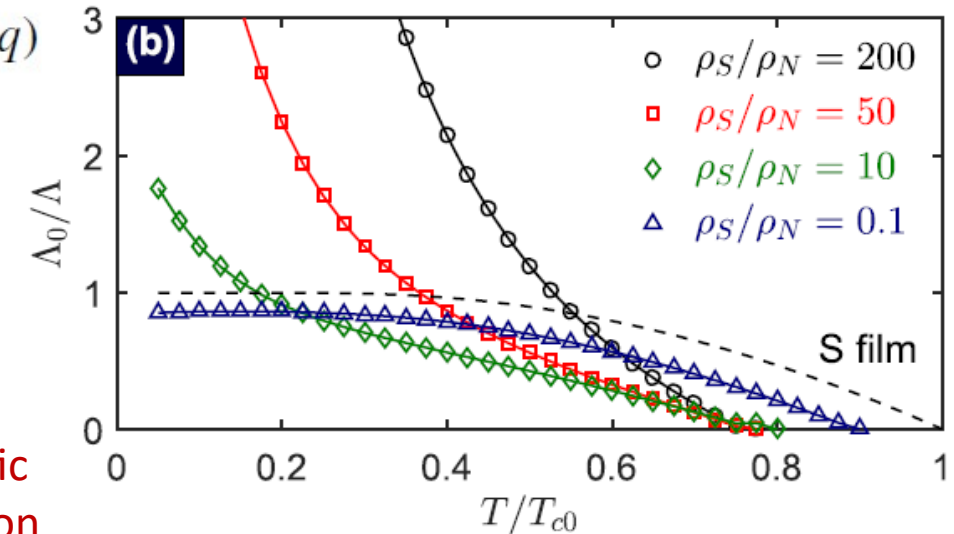
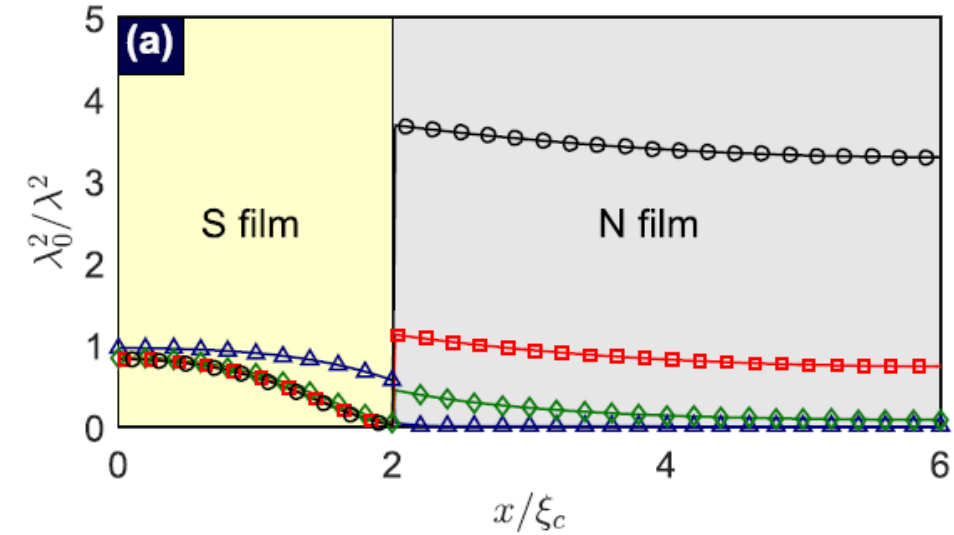
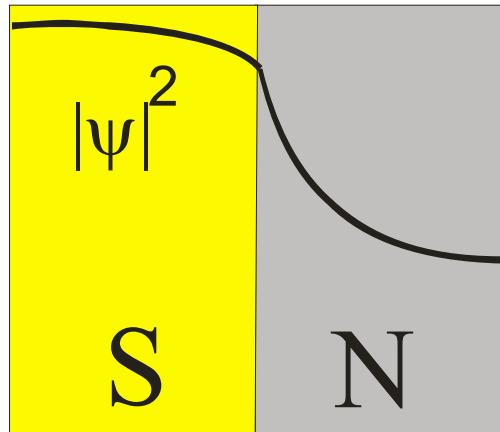


$$\mathbf{q} = \nabla\phi + 2\pi\mathbf{A}/\Phi_0$$

$$j_s(x, q) = -q \frac{c\Phi_0}{8\pi^2} \frac{1}{\lambda^2(x, q)}$$

$$\frac{1}{\lambda^2(x, q)} = \frac{16\pi^2 k_B T}{\hbar c^2} \times \frac{1}{\rho} \sum_{\omega_n \geq 0} \sin^2 \Theta(x, q)$$

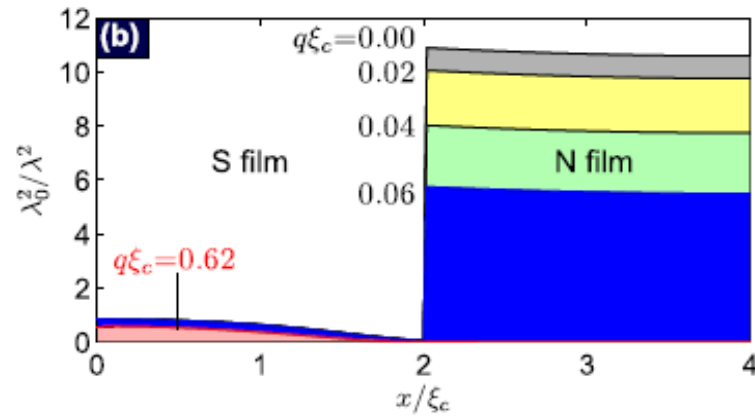
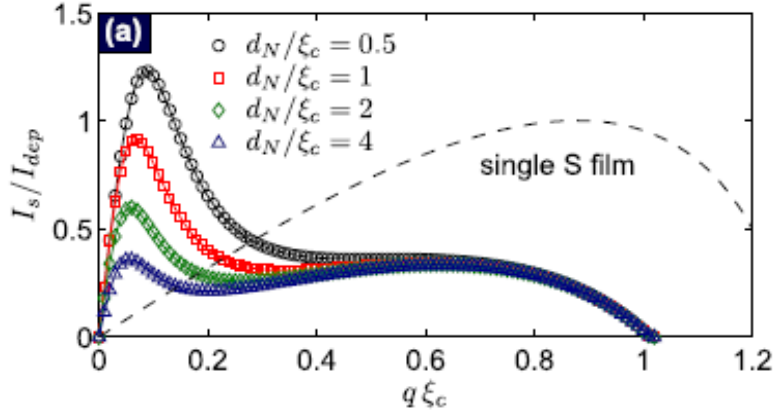
$$\Lambda^{-1}(q) = \int_0^{d_S+d_N} \frac{dx}{\lambda^2(x, q)}$$



Dominant contribution of N layer to diamagnetic response at low T - temperature driven transition to type I superconductor ($\xi > \lambda$)?

Transport properties of SN bilayer

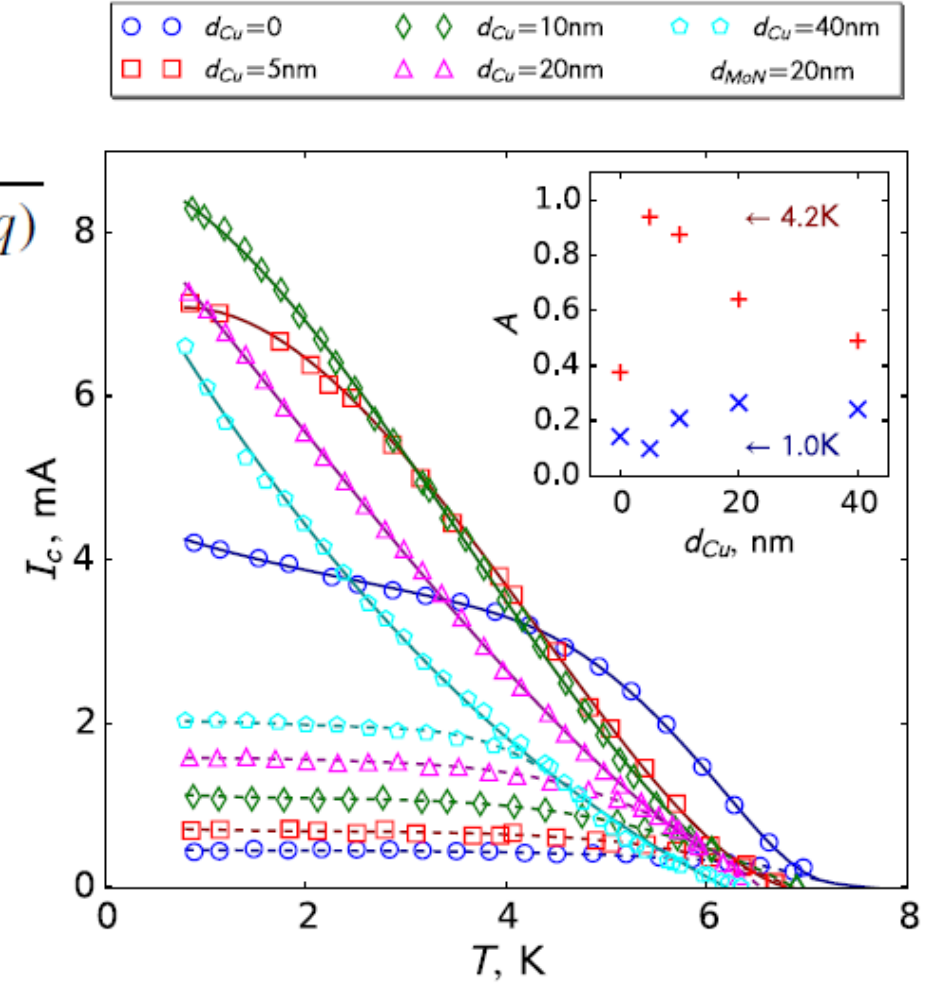
$$D_N/D_S = 200, d_S/\xi_c = 2, T/T_{c0} = 0.2$$



$$j_s(x, q) = -q \frac{c\Phi_0}{8\pi^2} \frac{1}{\lambda^2(x, q)}$$

I_c of SN bilayer is larger than I_c of single S layer

$$\rho_S/\rho_N = 12-60, \text{ MoN}(20\text{nm})/\text{Cu}$$

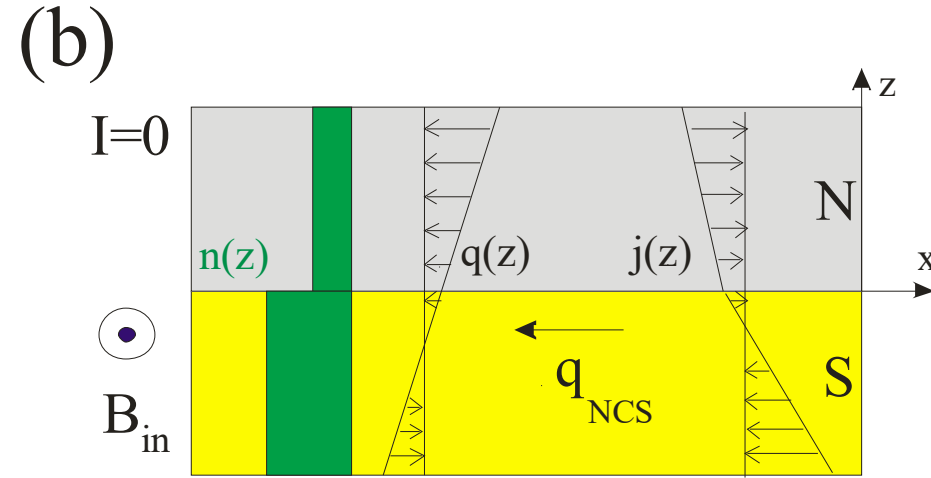
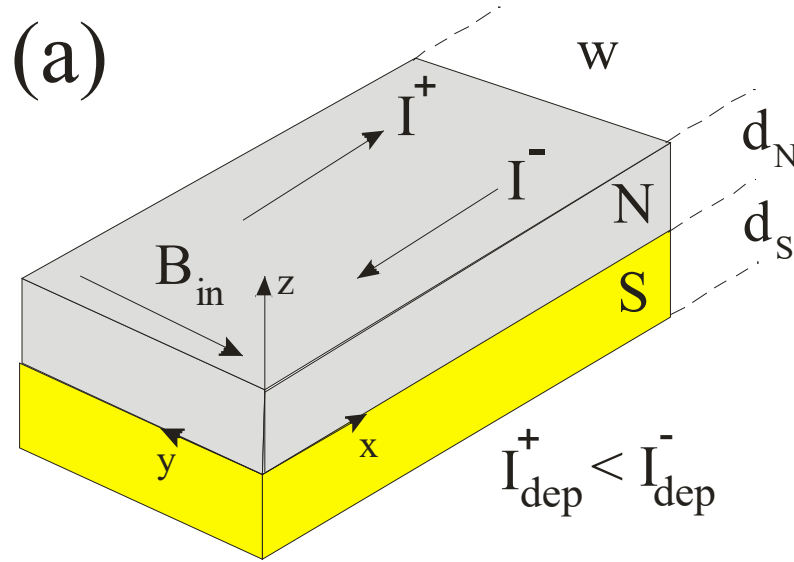


$$\hbar D \frac{\partial^2 \Theta}{\partial x^2} - \left(2\hbar\omega_n + \frac{D}{\hbar} q^2 \cos\Theta \right) \sin\Theta + 2\Delta \cos\Theta = 0$$

$$\Delta \ln\left(\frac{T}{T_{c0}}\right) + 2\pi k_B T \sum_{\omega_n \geq 0} \left(\frac{\Delta}{\hbar\omega_n} - \sin\Theta \right) = 0 \quad \hbar\omega_n = \pi k_B T (2n + 1)$$

Finite momentum state and diode effect in SN hybrid

Zero spin-orbit coupling



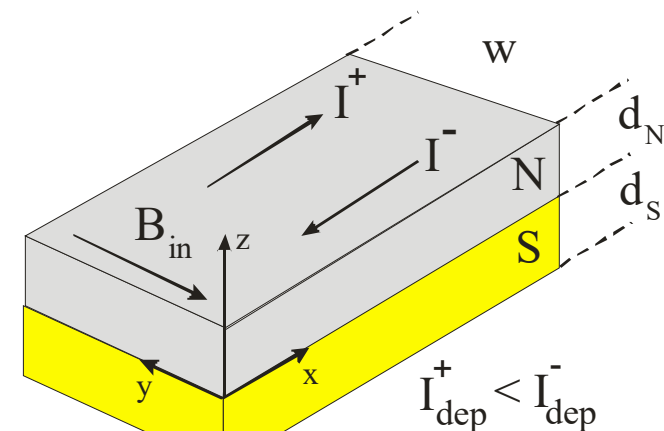
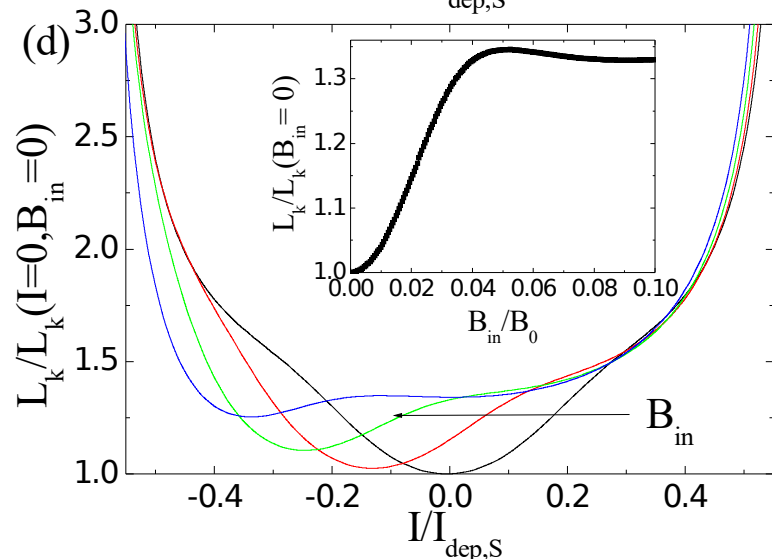
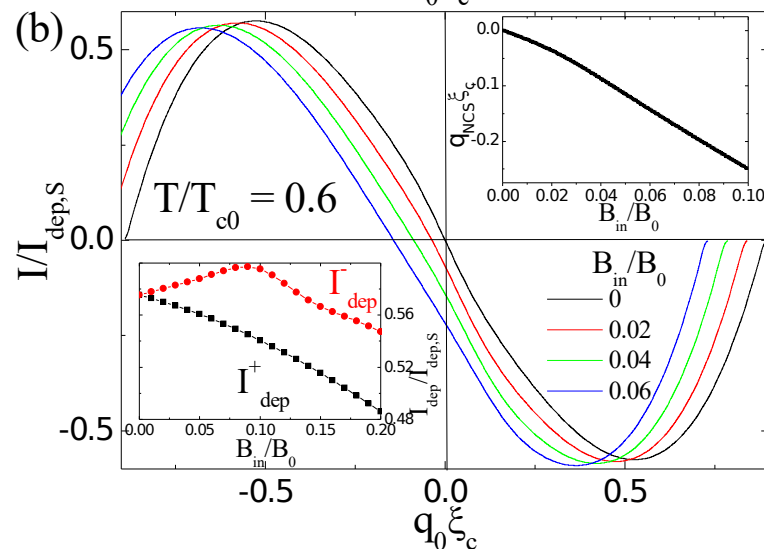
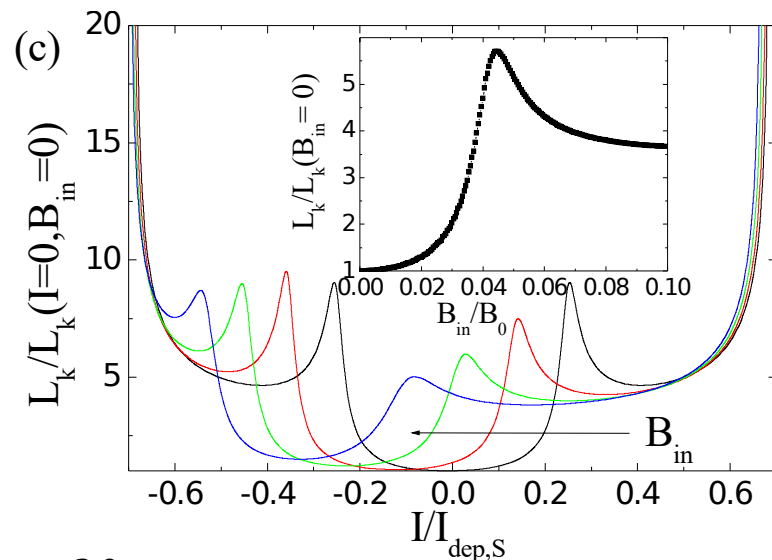
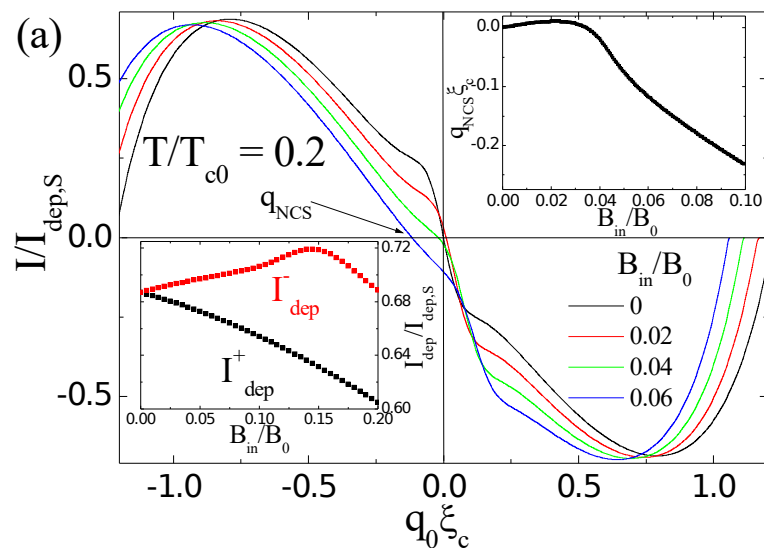
$$q(z) = \nabla\varphi + 2\pi A/\Phi_0 = q_0 + 2\pi B_{\text{in}} z/\Phi_0$$

$$j(z) = en(z)q(z) \quad n(z) \sim D/\lambda^2$$

$$I = \int j(z) dz = 0 \quad \langle q \rangle = \int q(z) dz = q_0 \neq 0$$

Particular direction – $\mathbf{q}_{\text{NCS}} \sim [\nabla n, \mathbf{B}_{\text{in}}]$.

Theoretical results (1D Usadel model)



$$d_N = d_S = 4\xi_c, \quad \xi_c = (\hbar D_S / k_B T_c)^{1/2}$$

$$D_N / D_S = 100$$

$$V = \frac{L_k}{c^2} \frac{dI}{dt} = \frac{l\hbar}{2e} \frac{dq}{dt}$$

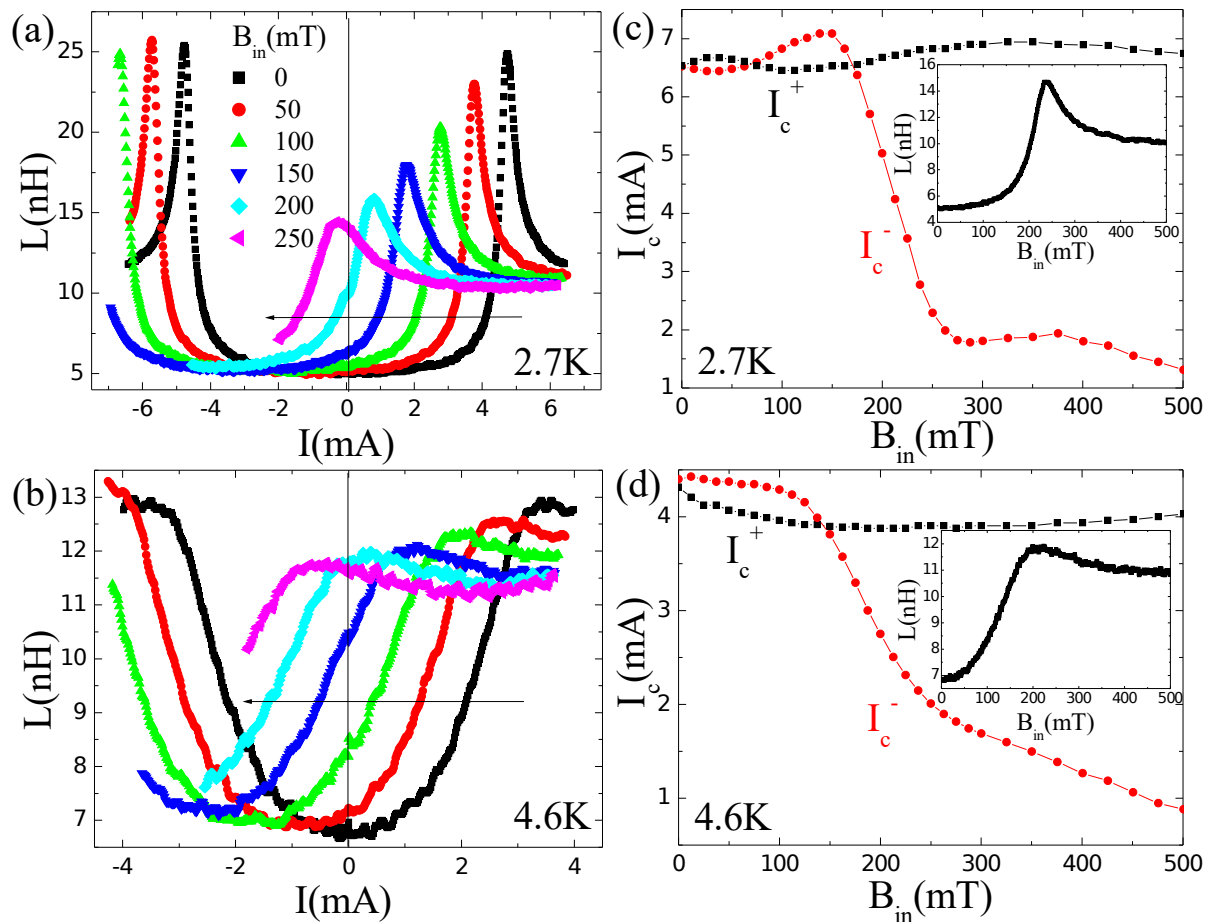
$$L_k = -l\hbar c^2 (dI/dq)^{-1} / 2|e|$$

Asymmetric $I(q) \neq -I(-q)$, diode effect $I_c^+ \neq I_c^-$ and nonreciprocal $L_k(I) \neq L_k(-I)$

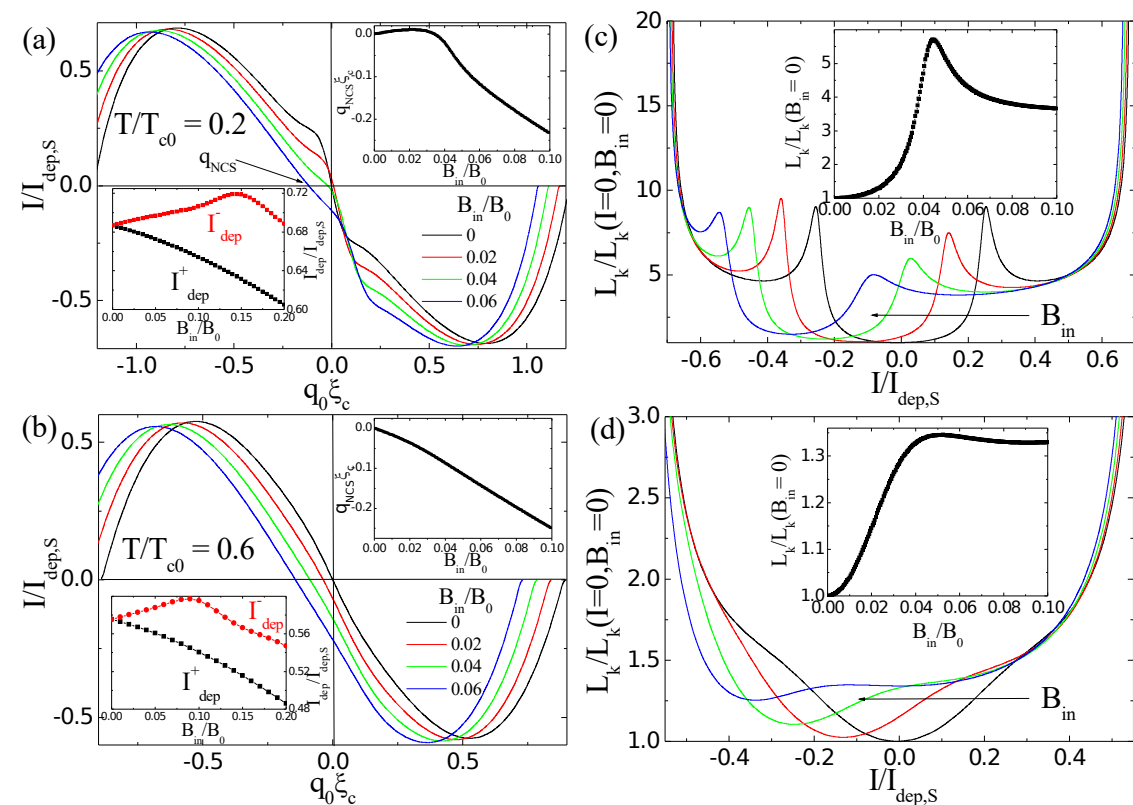
$$E_k = \int L_k(I) IdI / c^2 \rightarrow E_k = \int m(v) v dv$$

Experimental results. Nonreciprocal L_k and diode effect

Experiment



Theory



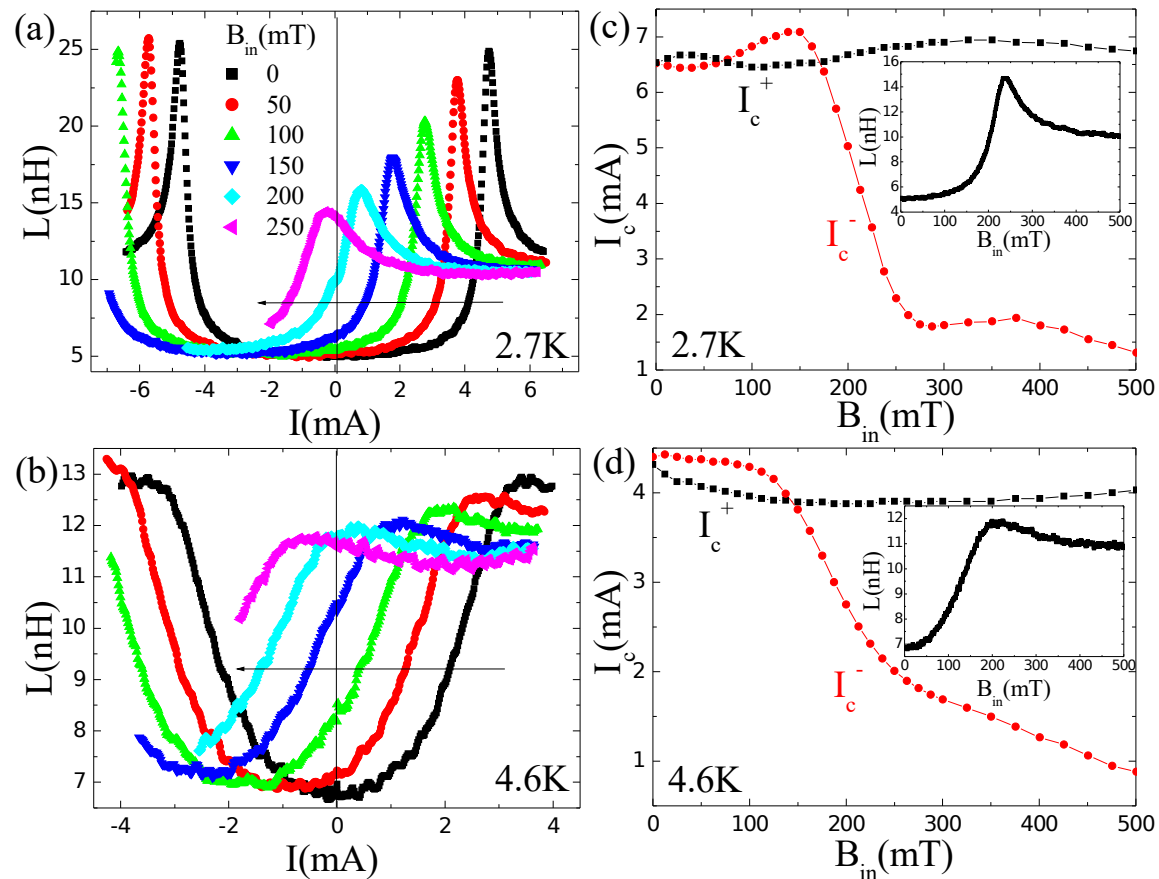
MoN(40nm)/Cu(40nm), $w = 4 \mu\text{m}$, $L = 3 \text{ mm}$ and $100 \mu\text{m}$, $T_c \sim 7.8 \text{ K}$

$$L = L_g + L_k \quad (L_g \sim 4.7 \text{ nH})$$

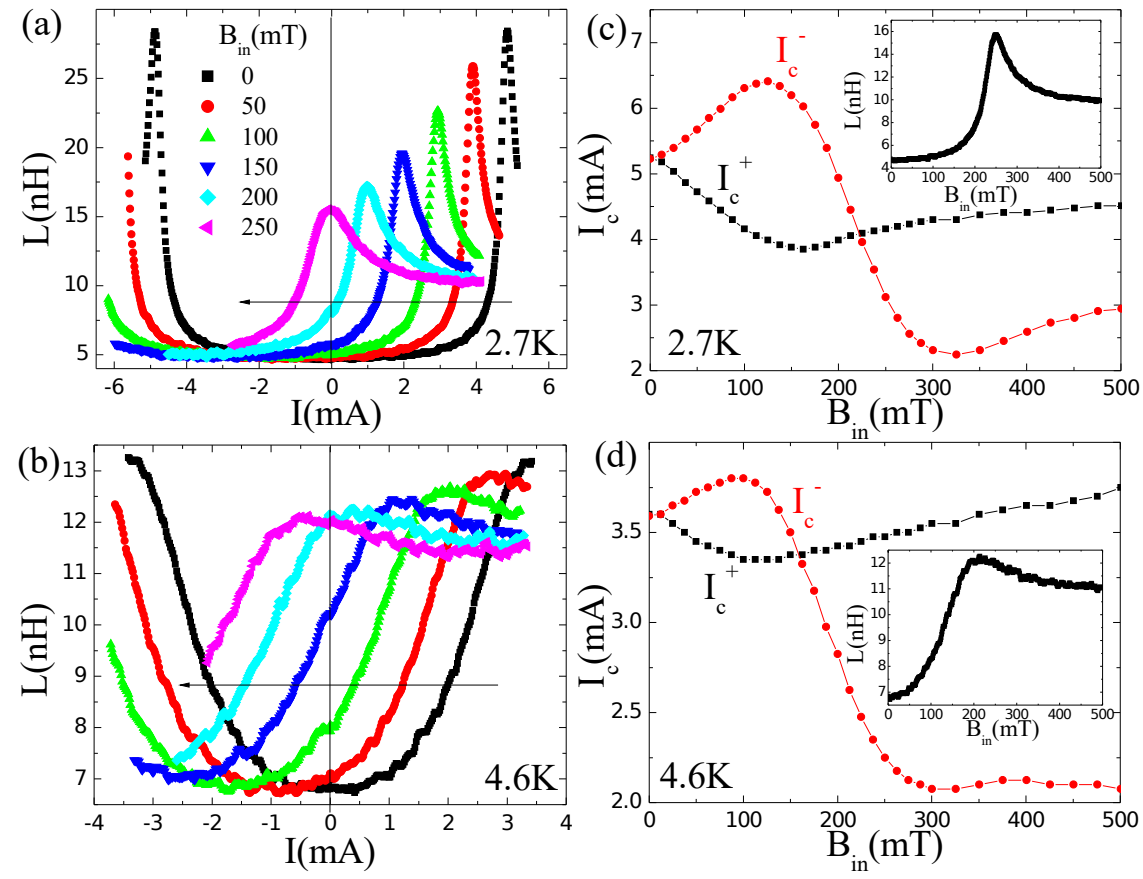
Unexpected sign reversal of SDE at large B_{in} and its giant value !

Experimental results. Nonreciprocal L_k and diode effect

Sample A2



Sample A4

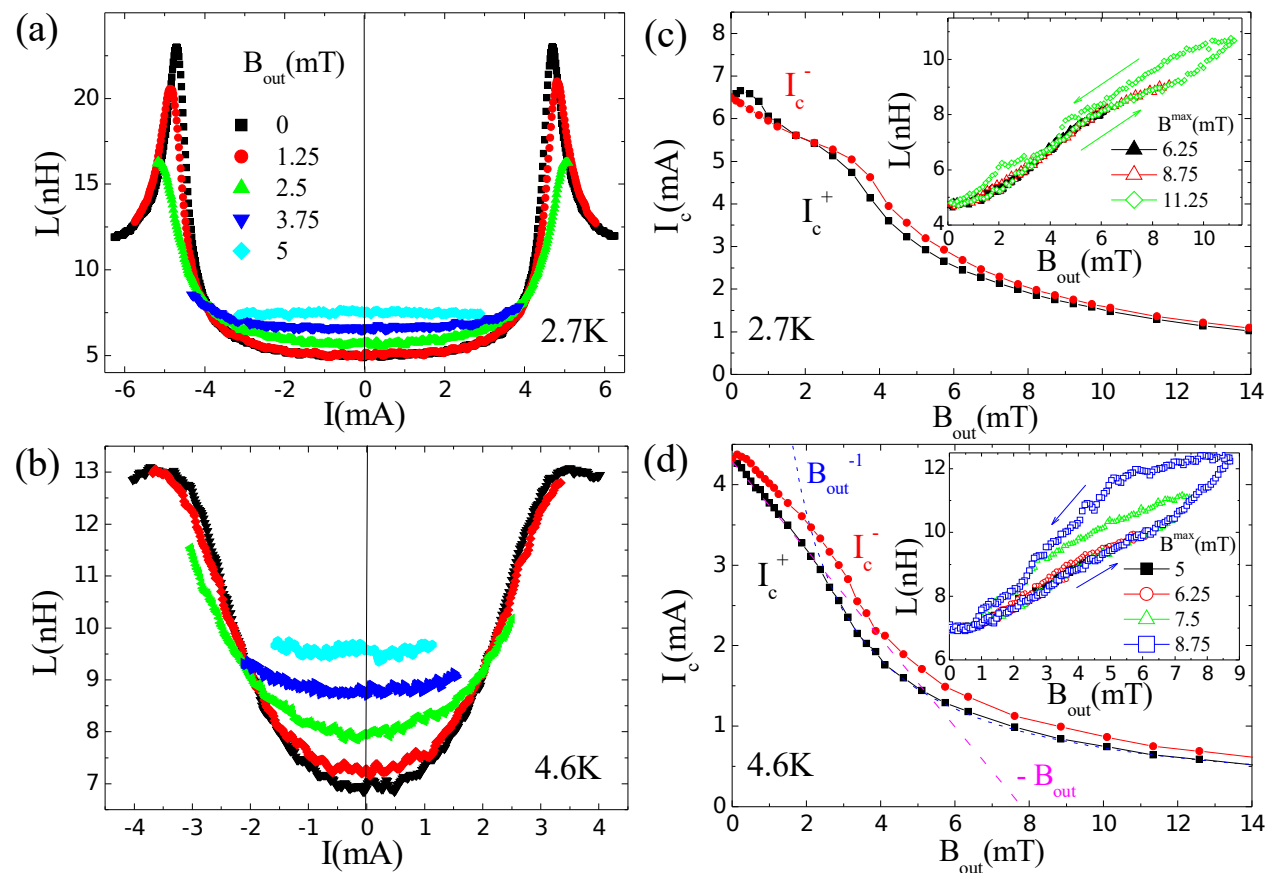
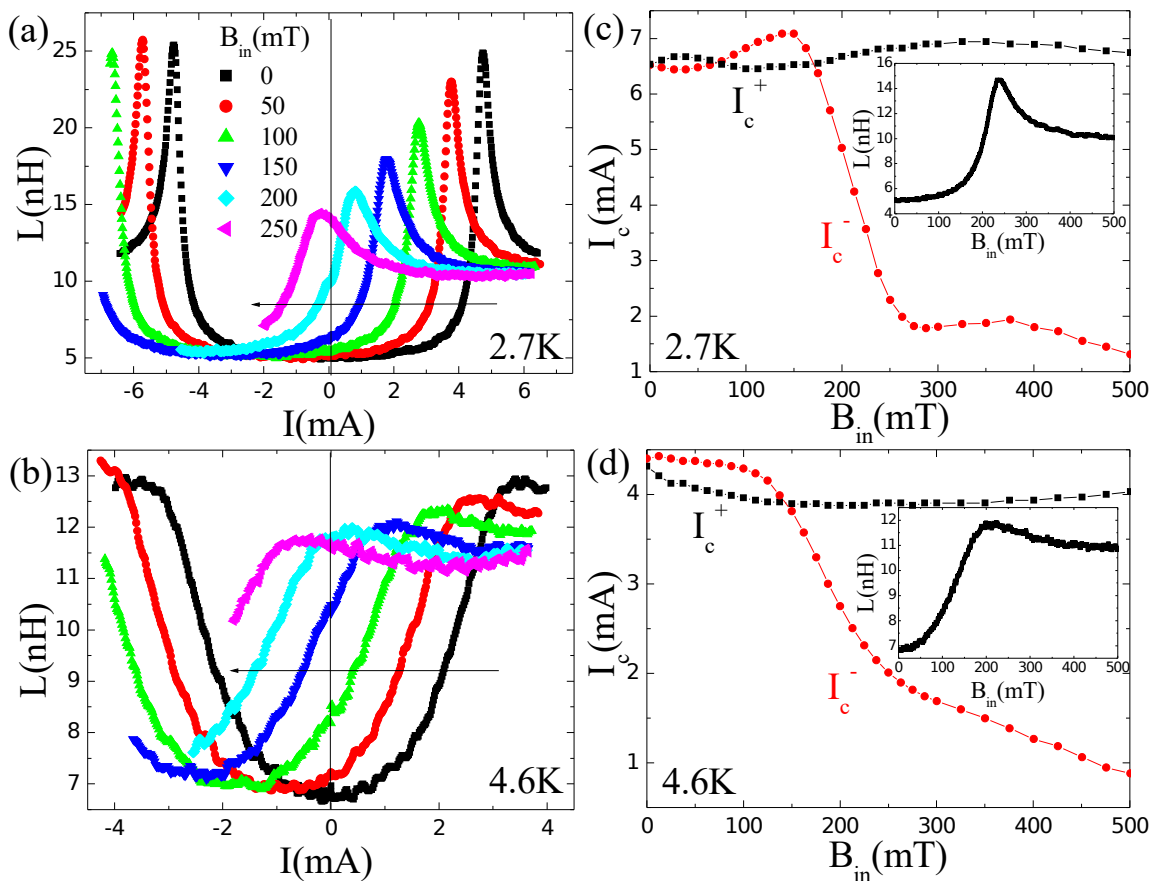


Unexpected sign reversal of SDE at large B_{in} and its giant value !

Experimental results. Nonreciprocal L_k and diode effect

In-plane field

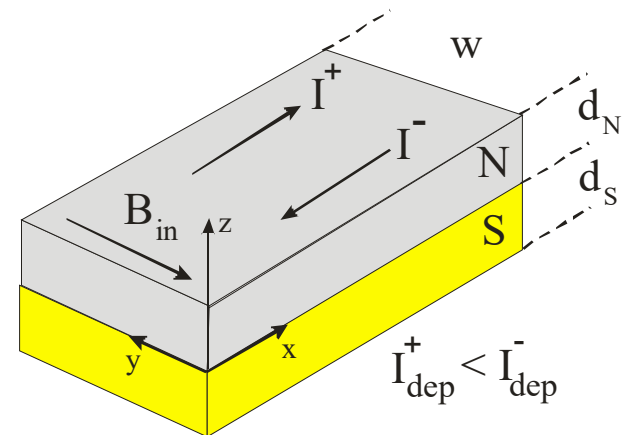
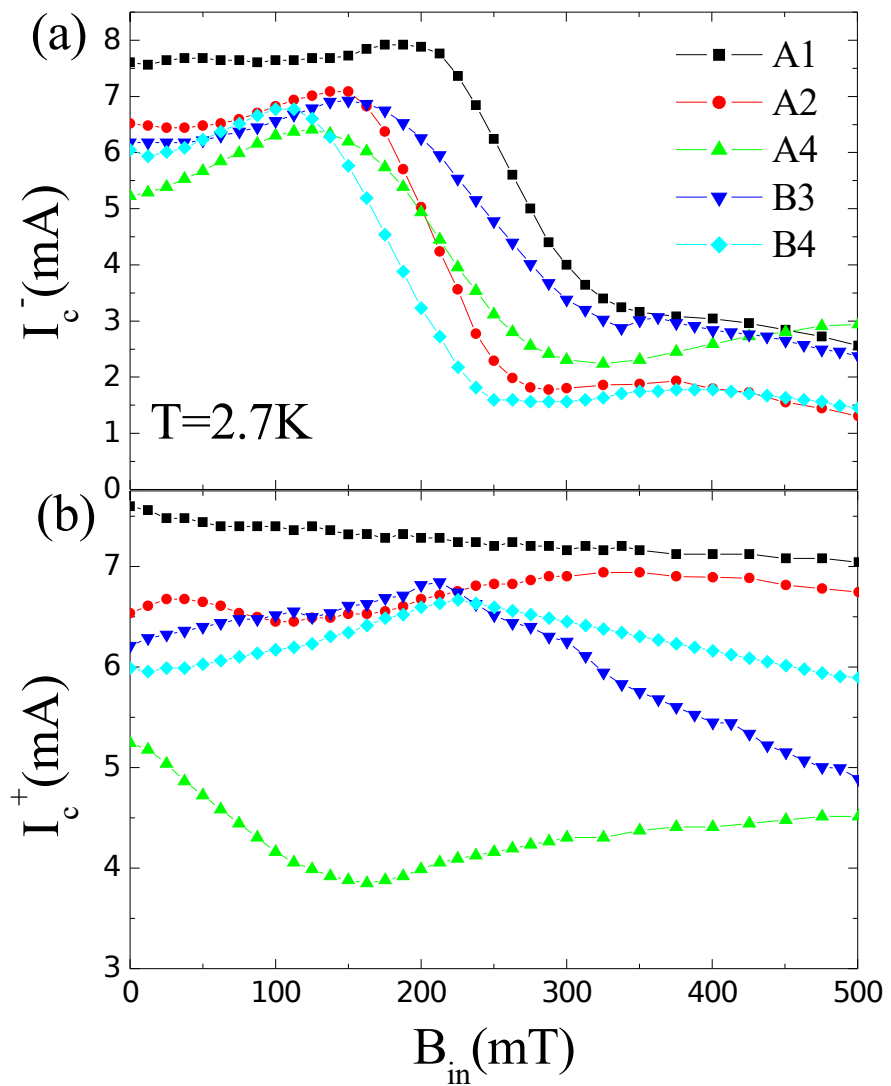
Out-of-plane field



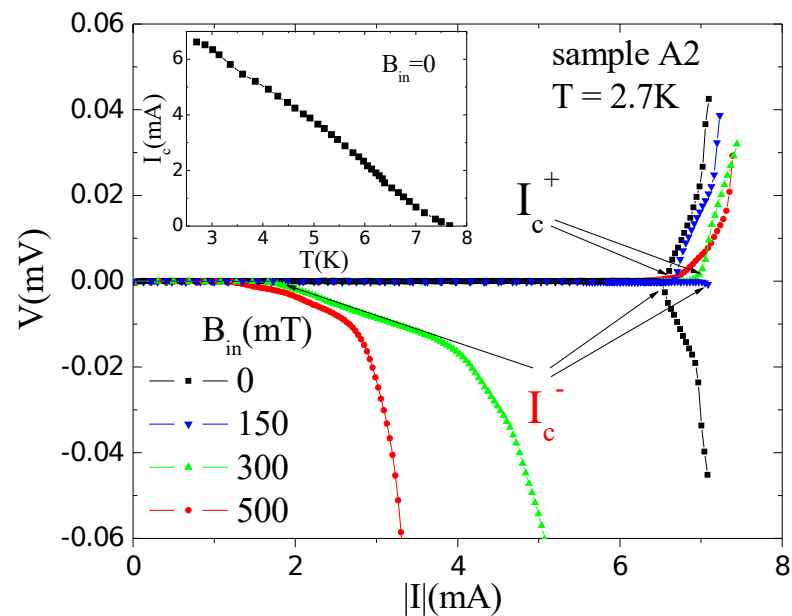
Sample-dependent local defects lead to sample-dependent $I_c(B_{in})$.

L_k is not sample-dependent, it is characteristic of whole strip.

Possible origin of sign change of diode effect at large B_{in}

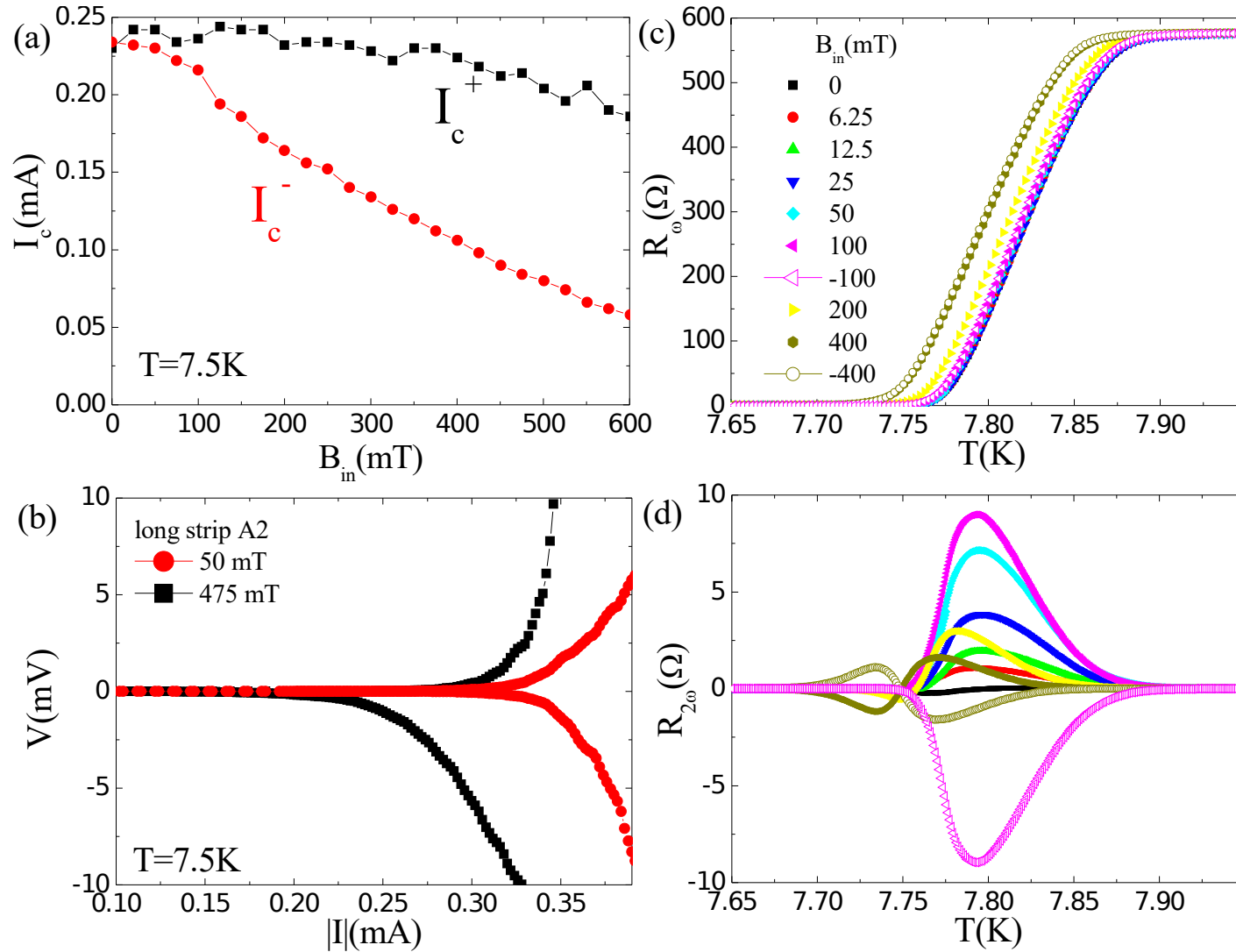


Entry of in-plane vortices at $B \sim B_{c1} = \Phi_0 / (d_S + d_N)^2 \sim 320$ mT ?



Diode effect and nonreciprocal resistance near T_c

Sample A2



1) $R \sim \exp(-dF(I/I_c)/k_B T)$

2) Nonreciprocal viscosity of vortex motion $\eta(I) \neq \eta(-I)$

Relation with existing experiments

Nature 2020

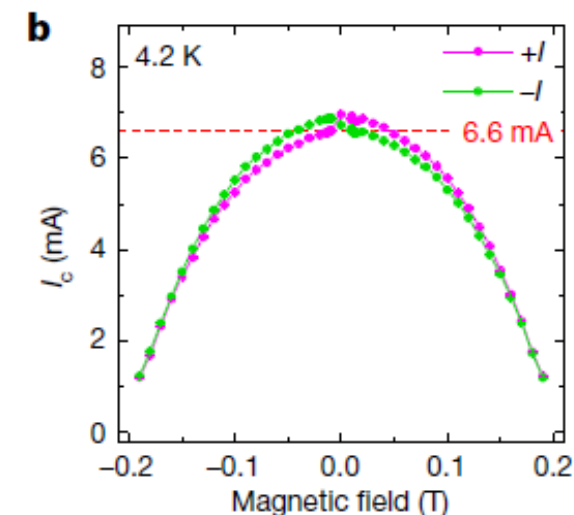
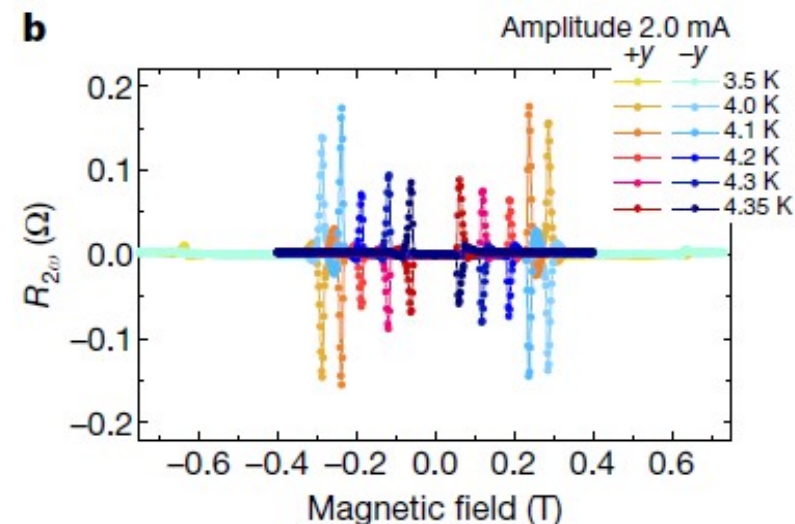
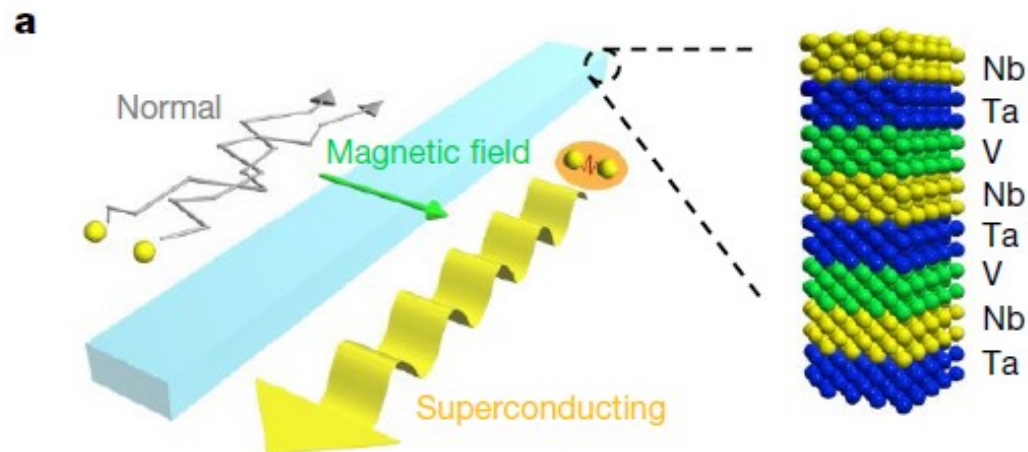
Article

Observation of superconducting diode effect

<https://doi.org/10.1038/s41586-020-2590-4>

Fuyuki Ando¹, Yuta Miyasaka¹, Tian Li¹, Jun Ishizuka², Tomonori Arakawa^{3,4}, Yoichi Shlota¹, Takahiro Moriyama¹, Youichi Yanase² & Teruo Ono^{1,4}✉

Received: 14 March 2020



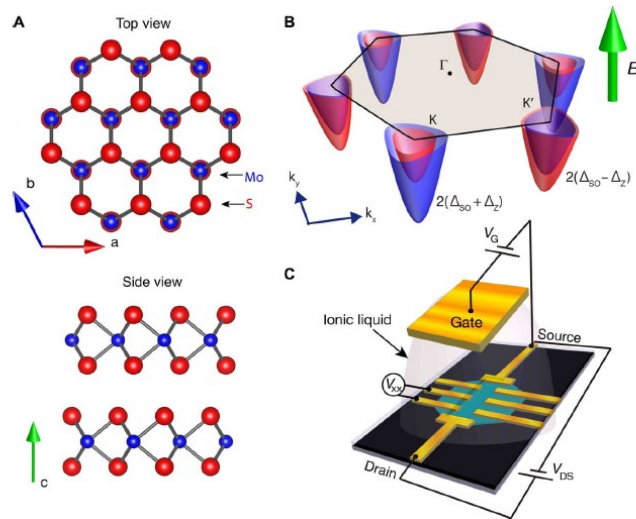
Main explanation: spin-orbit coupling as an origin of the effect

No measurements of L_k and diode effect in out-of-plane field

CONDENSED MATTER PHYSICS

Nonreciprocal charge transport in noncentrosymmetric superconductors

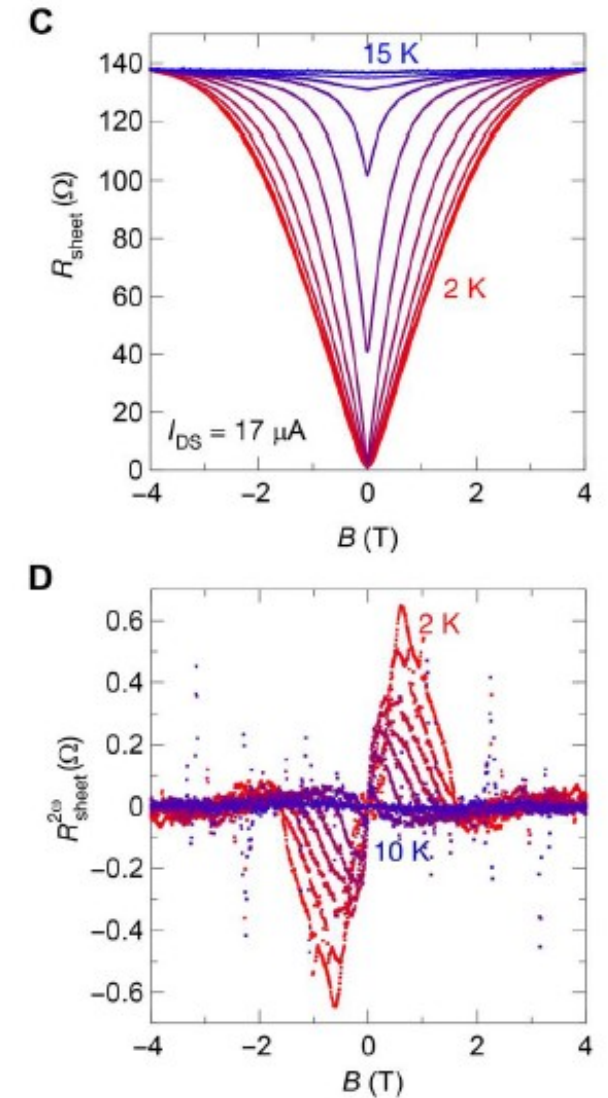
Ryohei Wakatsuki,^{1*} Yu Saito,^{1*} Shintaro Hoshino,² Yuki M. Itahashi,¹ Toshiya Ideue,¹ Motohiko Ezawa,¹ Yoshihiro Iwasa,^{1,2} Naoto Nagaosa^{1,2†}



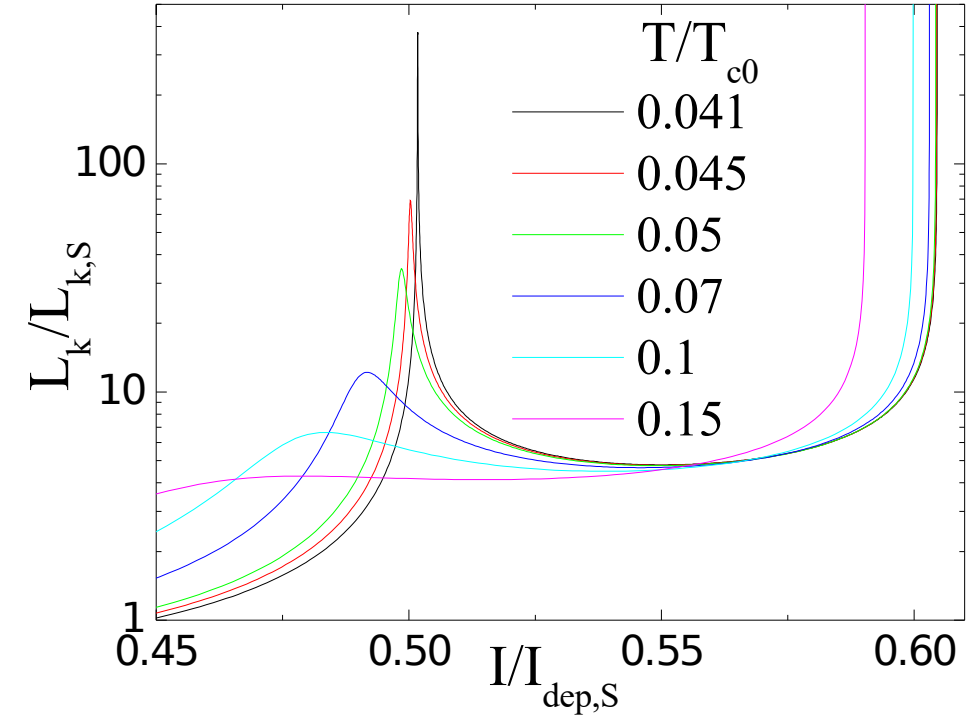
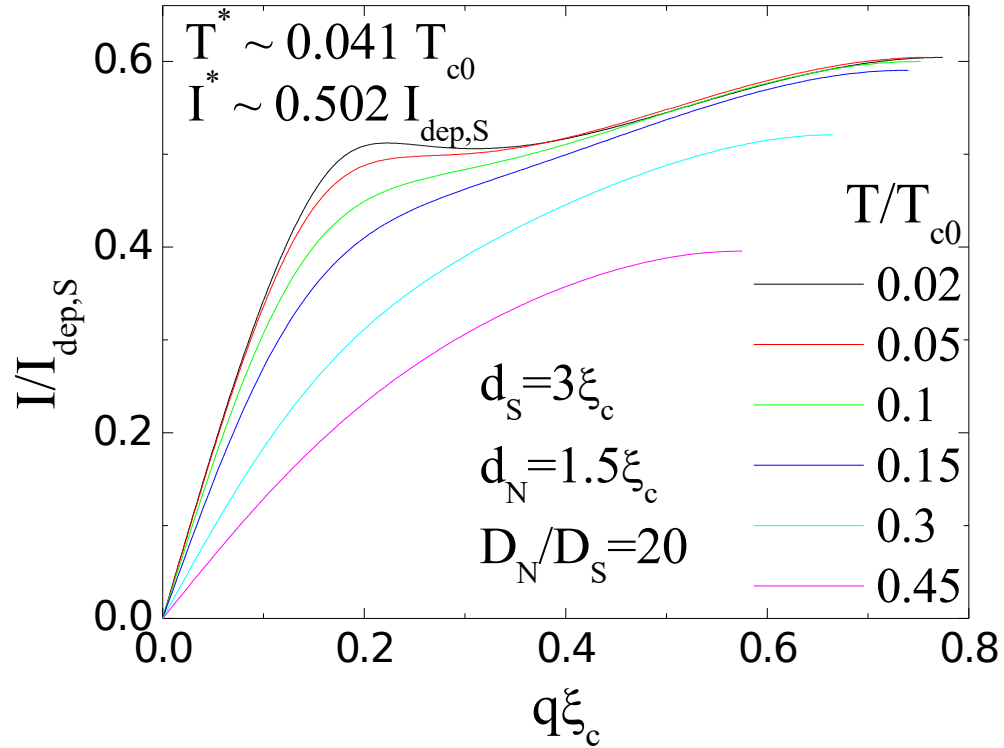
d=20 nm

MoS₂ multilayer flake with the 2H polytype, in which the two adjacent layers are rotated by π with respect to each other, therefore making the whole structure centrosymmetric in contrast to the noncentrosymmetric monolayer (Fig. 2A). However, once the gate voltage is applied, the electric field breaks the out-of-plane inversion symmetry, making the neighboring monolayers inequivalent. At the same time, the conduction band minimum shifts to the $\pm K$ points, where interlayer coupling is very weak, and thus electrons are quite localized within each layer. Consequently, the in-plane inversion asymmetry in each

Main explanation: spin-orbit coupling as an origin of the effect



Unique nonlinear kinetic inductance of SN hybrid

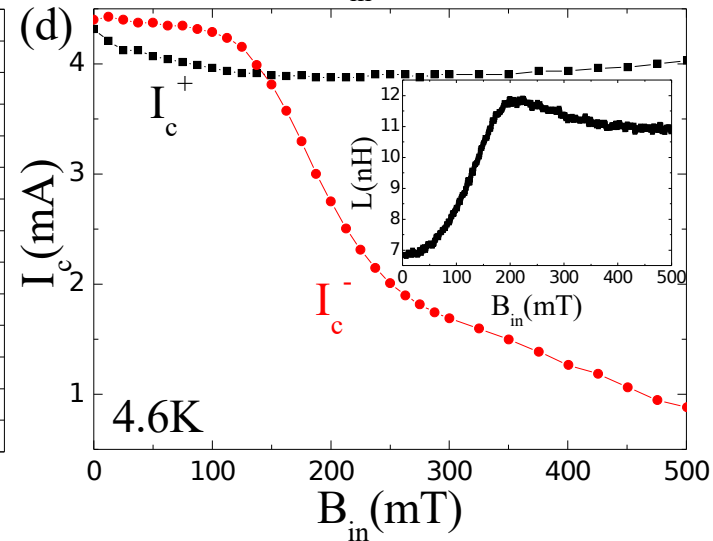
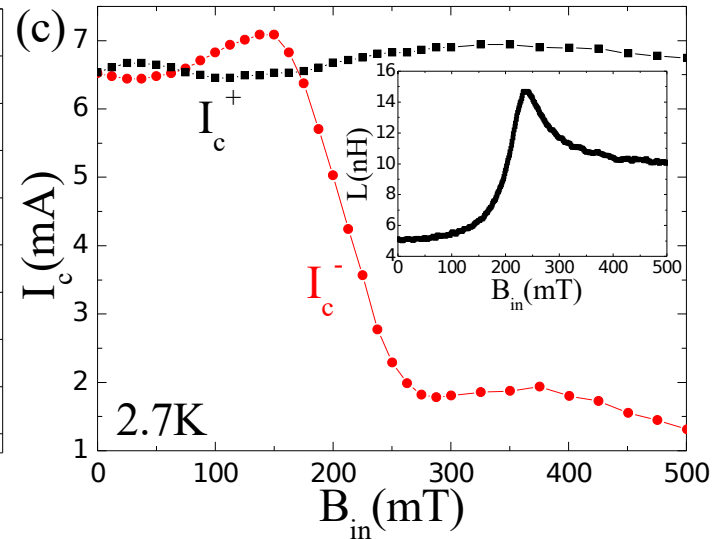
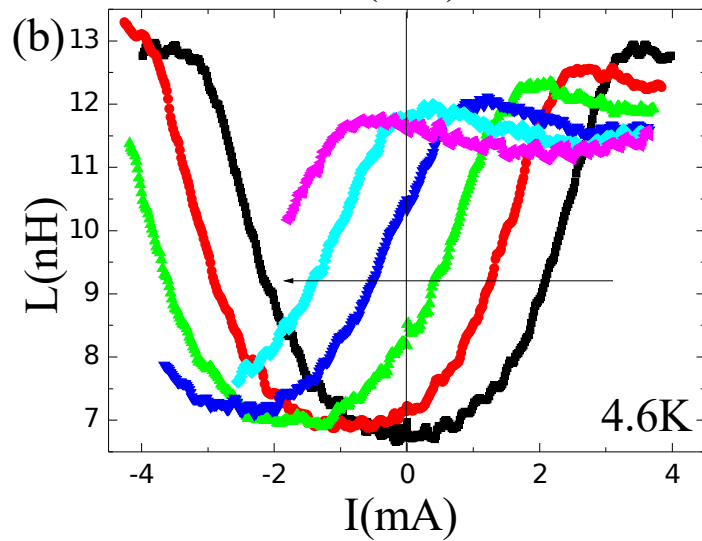
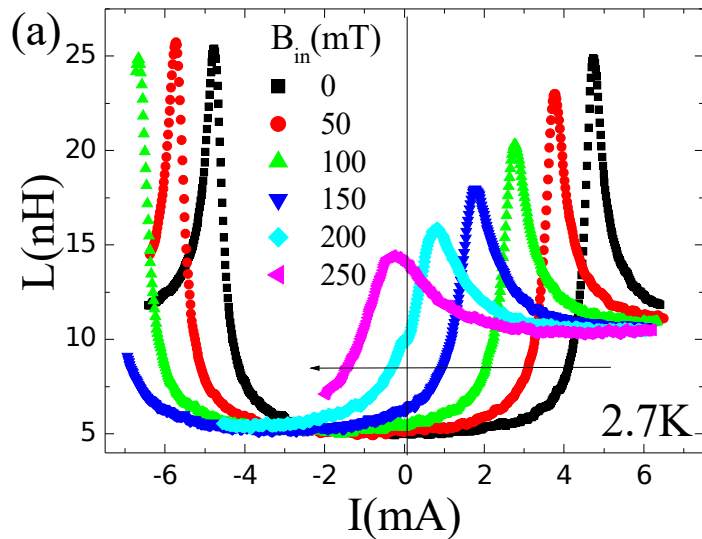


Maximum of $I(q)$ at low q – suppression of proximity induced superconductivity in N layer.

$$L_k = -l\hbar c^2 (dI/dq)^{-1} / 2|e|$$

$$\hbar D \frac{\partial^2 \Theta}{\partial x^2} - \left(2\hbar \omega_n + \frac{D}{\hbar} q^2 \cos \Theta \right) \sin \Theta + 2\Delta \cos \Theta = 0$$

Experimental observation of peak on $L_k(I)$



**MoN(40nm)/Cu(40nm), $w = 4 \mu\text{m}$,
 $L = 3 \text{ mm}$, $T_c \sim 7.8 \text{ K}$**

$$d_s = 6\xi_c$$

$$d_N = 6\xi_c$$

$$T^* = 0.025 T_{c0} \sim 200 \text{ mK}$$

M. Yu. Levichev, I. Yu. Pashenkin,
N. S. Gusev, D. Yu. Vodolazov, *Finite momentum
superconductivity in superconducting hybrids:
Orbital mechanism*, PRB (2023).

Magnetometer

NbN

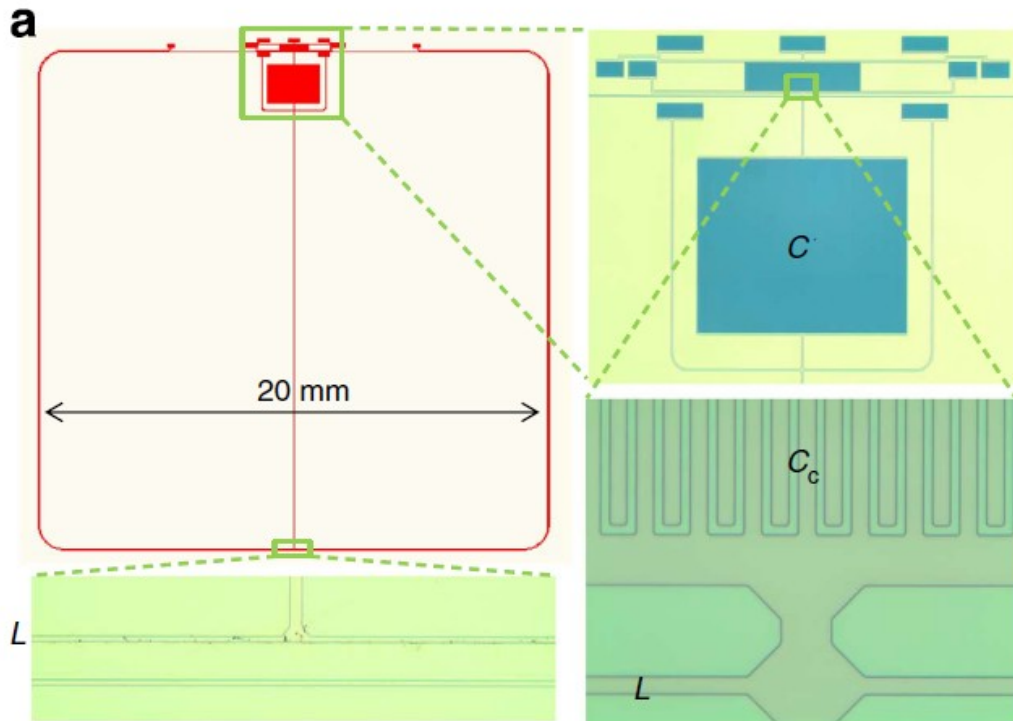
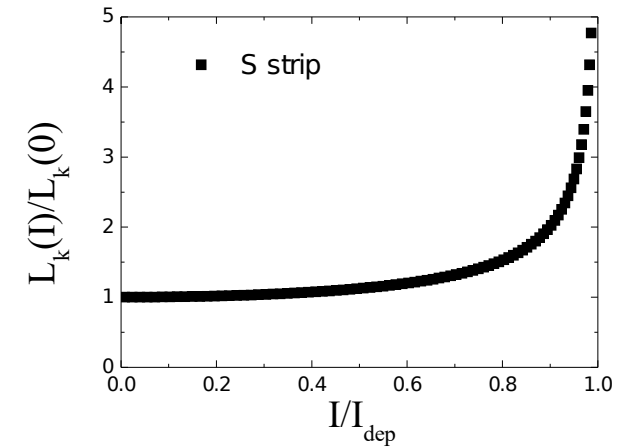
ARTICLE

Received 10 Jan 2014 | Accepted 1 Aug 2014 | Published 10 Sep 2014

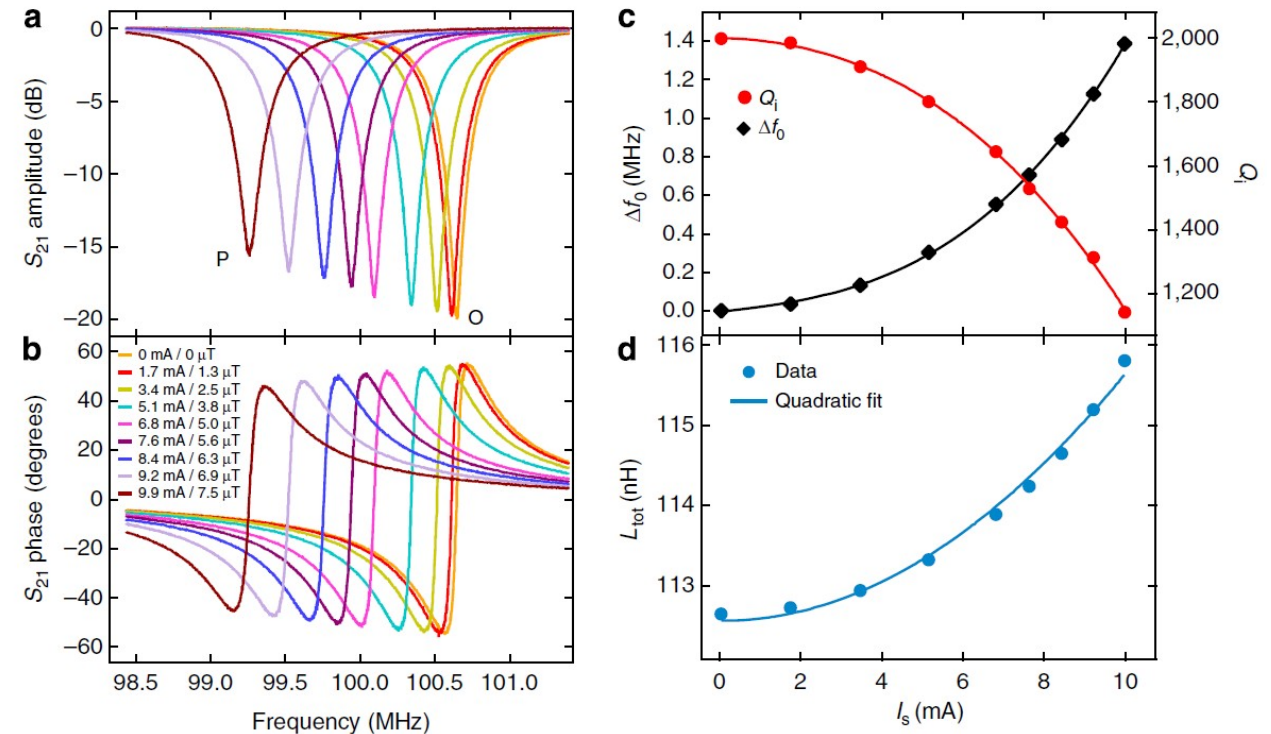
DOI: 10.1038/ncomms5872

Kinetic inductance magnetometer

Juho Luomahaara¹, Visa Vesterinen¹, Leif Grönberg¹ & Juha Hassel¹



LC contour with resonance frequency $\sim 1/(LC)^{1/2}$



Parametric amplifiers, up-converters, etc.

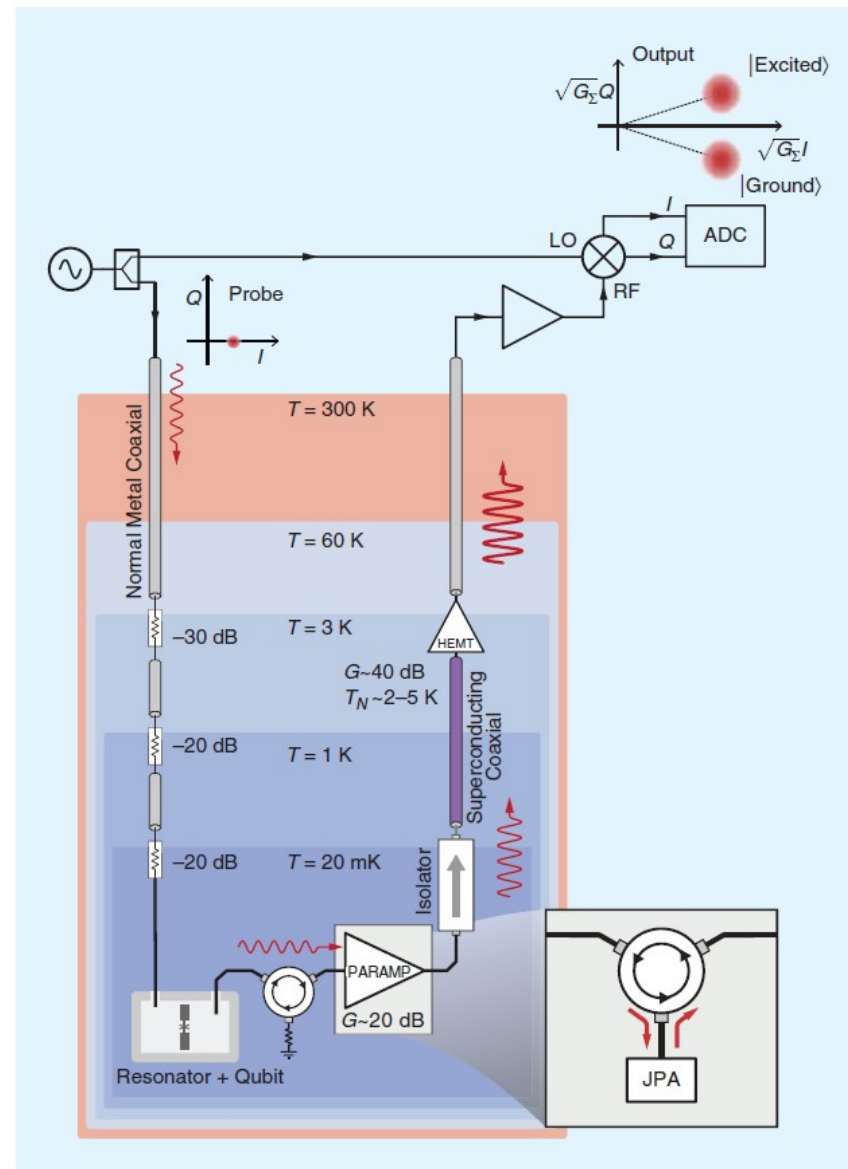
$L_k(I) \sim L_{k0} (1 + I^2/I_*^2)$ – similarity with Kerr nonlinearity $n(E) \sim n_0(1 + E^2/E_*^2)$

$$V = \frac{d}{dt}(L_k I) = L_{k0} \frac{dI}{dt} + 3L_{k0} \frac{I^2}{I_*^2} \frac{dI}{dt}$$

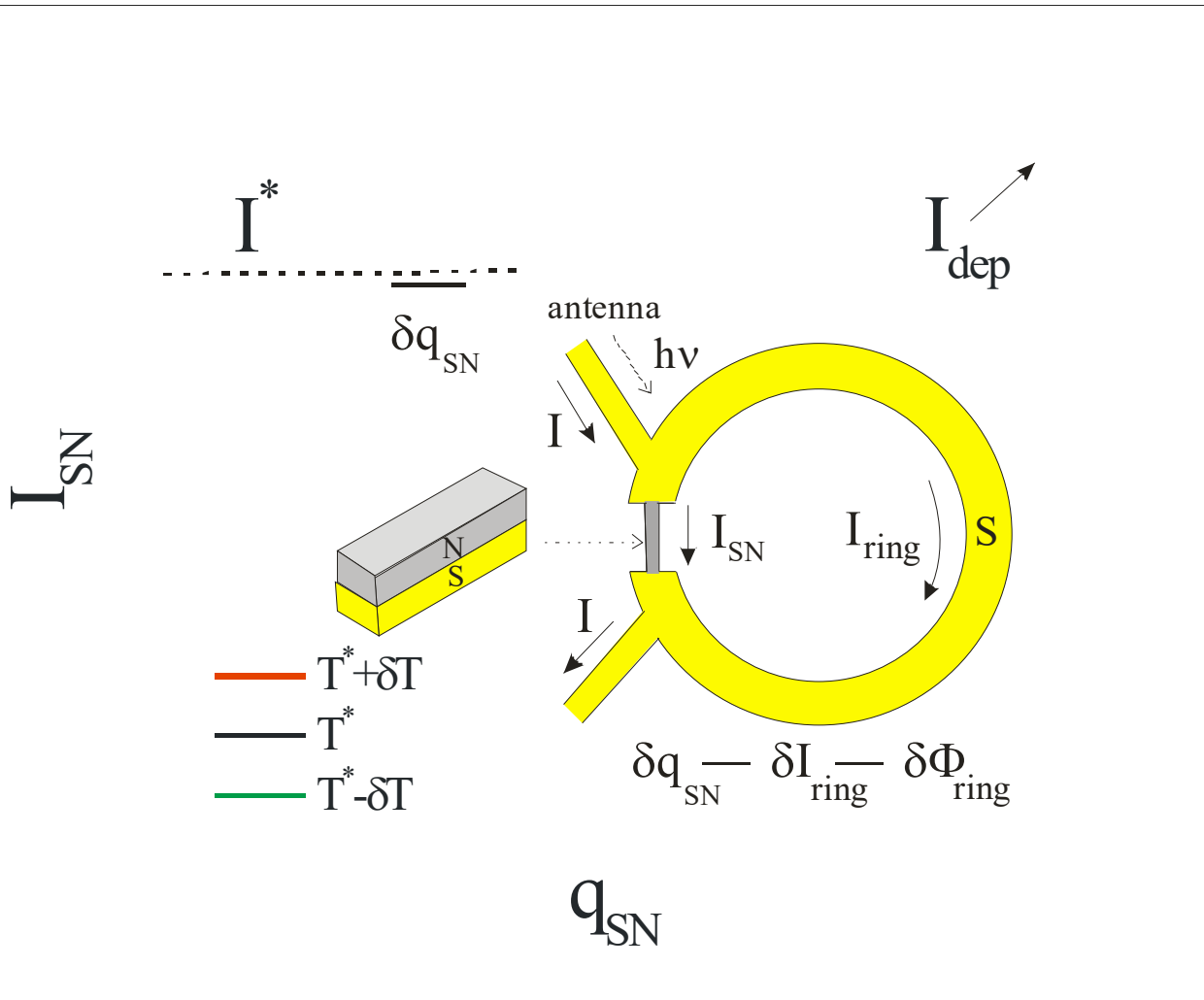
$$I = I_0 + I_c \cos(\omega_c t) + I_s \cos(\omega_s t)$$

$$\frac{dI}{dt} = -\omega_c I_c \sin(\omega_c t) - \omega_s I_s \sin(\omega_s t)$$

$$\begin{aligned} -I^2 \frac{dI}{dt} = & \omega_c I_0^2 I_c \sin(\omega_c t) + (\omega_c - \omega_s) I_0 I_c I_s \sin((\omega_c - \omega_s)t) \\ & + (\omega_c + \omega_s) I_0 I_c I_s \sin((\omega_c + \omega_s)t), \end{aligned}$$



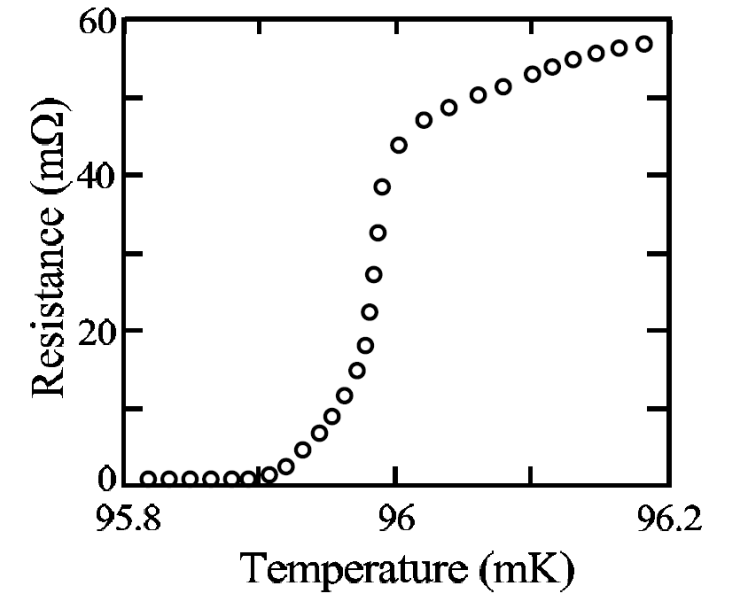
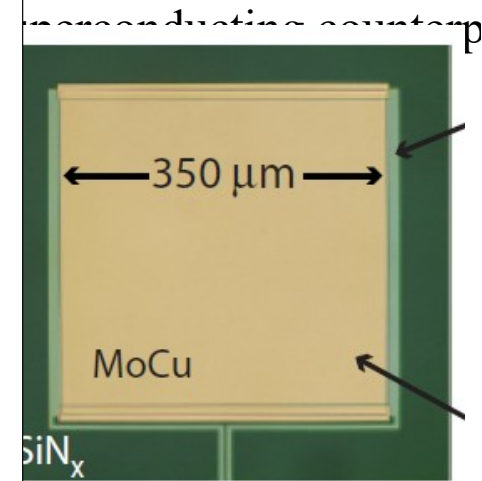
Nonlinear kinetic inductance sensor of single microwave photons



plateau on dependence $I_{\text{SN}}(q_{\text{SN}})$ in SN bridge at $T=T^*$ and $I=I^*$.

strong temperature dependence of $I_{\text{SN}}(q_{\text{SN}})$ near T^* and I^* .

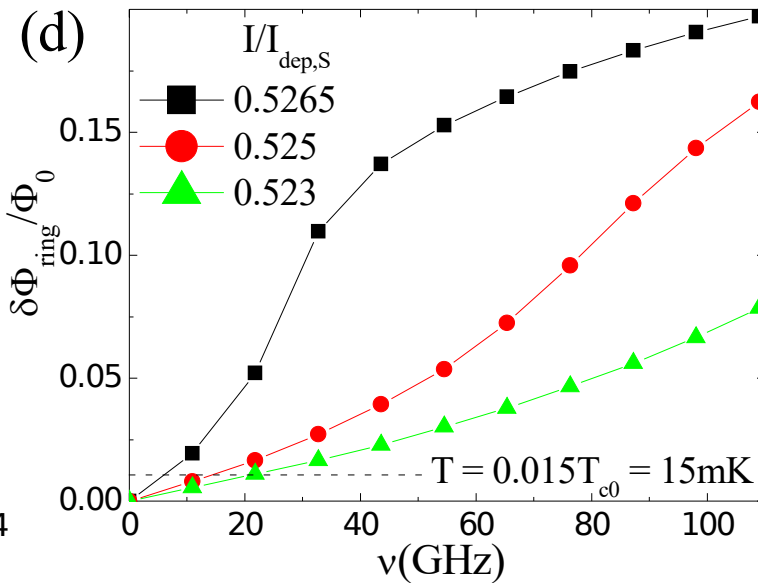
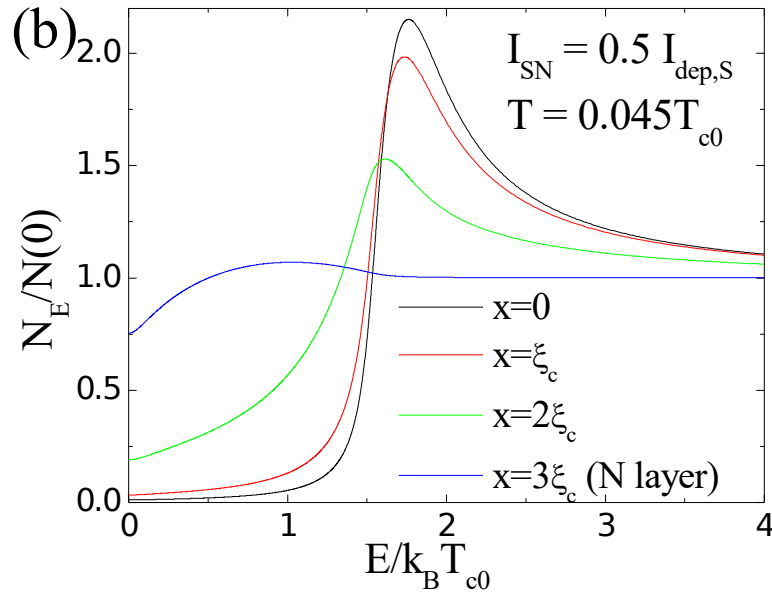
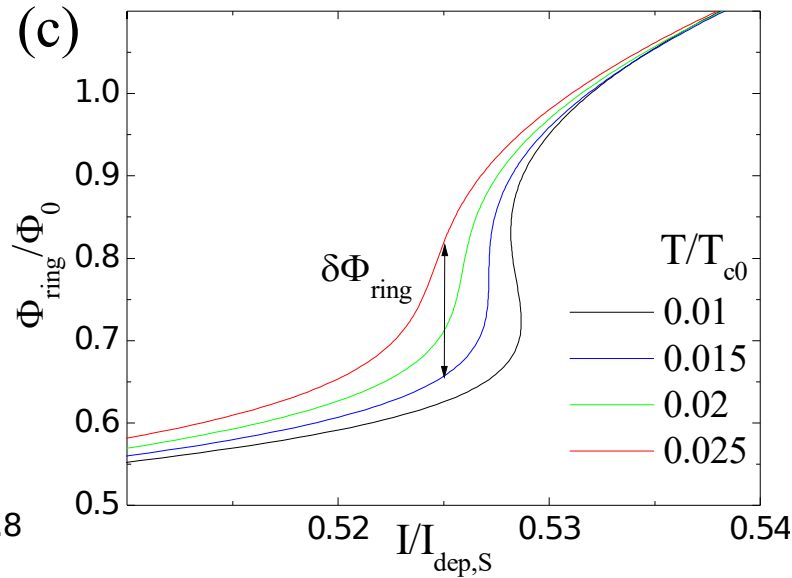
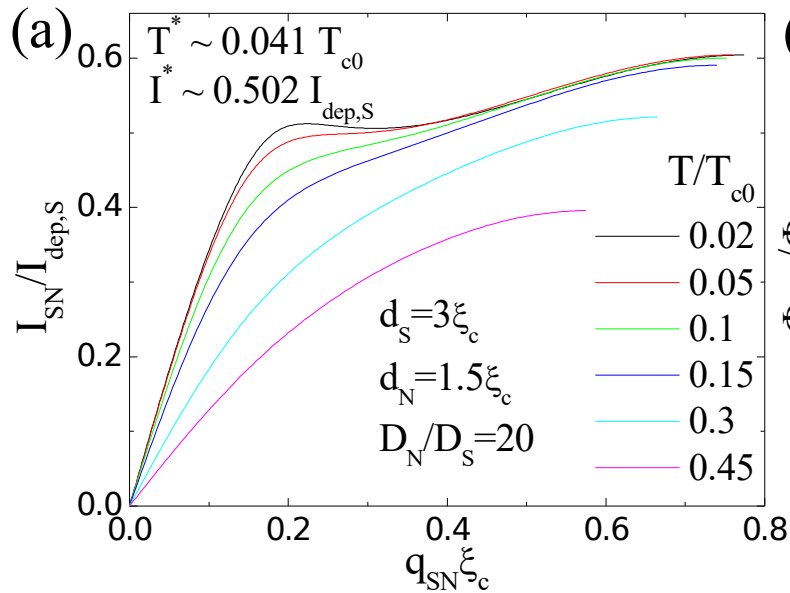
change of Φ in the ring is measured by SQUID.



Nonlinear kinetic inductance sensor (NKIS) – uses large dq/dI

Transition edge sensor (TES) – uses large dR/dT

Nonlinear kinetic inductance sensor of single microwave photons



$$I = I_{\text{SN}} + I_{\text{ring}} = I_{\text{SN}}(q_{\text{SN}}) + 1.55 \alpha q_{\text{SN}} \xi_c I_{\text{dep,S}}$$

$$\Phi_{\text{ring}} = L_G I_{\text{ring}}$$

$$\alpha = l_{\text{SN}} w_{\text{ring}} / l_{\text{ring}} w_{\text{SN}}$$

$$q_{\text{ring}} = q_{\text{SN}} l_{\text{SN}} / l_{\text{ring}}$$

$$l_{\text{SN}} = 1 \mu\text{m}, w_{\text{SN}} = 100 \text{ nm}$$

$$l_{\text{ring}} = 2.4 \text{ mm}, w_{\text{ring}} = 12 \mu\text{m}$$

$$\delta T = \hbar \nu / C_e(T) V_{\text{SN}} - \text{energy conservation}$$

$$T_{c0} = 1 \text{ K (10 K)}, D_s = 0.5 \text{ cm}^2/\text{s}$$

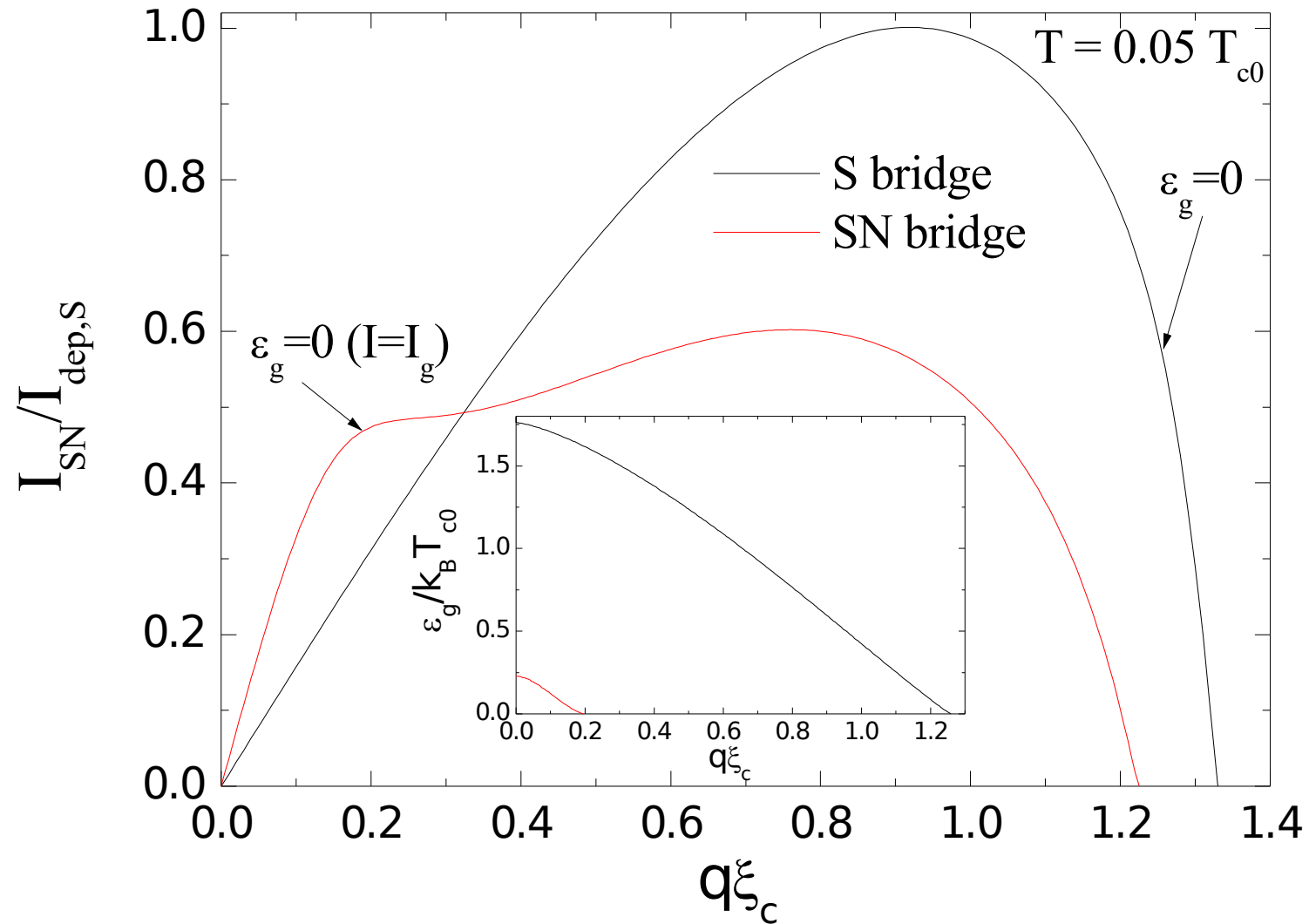
$$I_{\text{dep,S}} = 30.6 \mu\text{A (306 } \mu\text{A)}$$

$$\text{Noise} \rightarrow \delta\Phi_n \omega^{1/2} < \sim 10^{-3} \Phi_0$$

$$T = 15 \text{ mK (150 mK)} - 10 \text{ GHz (30 GHz)}$$

Gapless superconductivity in SN bridge

$$\varepsilon_g(I=I_{\text{dep}}) \sim \Delta_0/3 \text{ (Maki 1963)}$$



$$D_N/D_S = 20$$

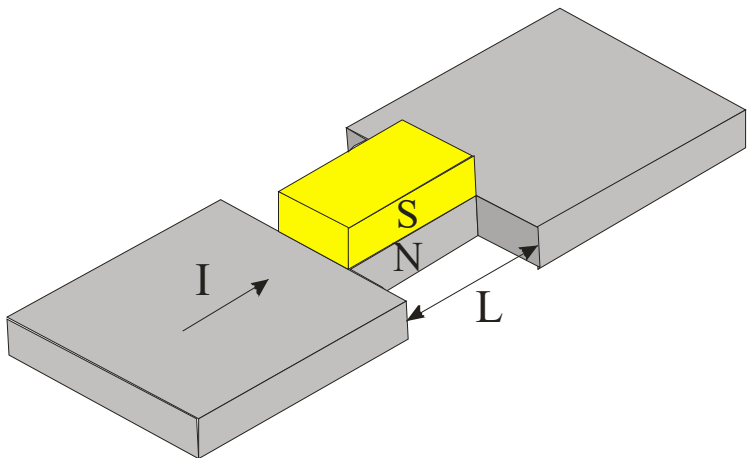
$$d_S = 3\xi_c$$

$$d_N = 1.5\xi_c$$

$$\xi_c = (\hbar D_N/k_B T_{c0})^{1/2}$$

$$q = \nabla\phi + 2\pi A/\Phi_0$$

Nascent vortices in SN bridge

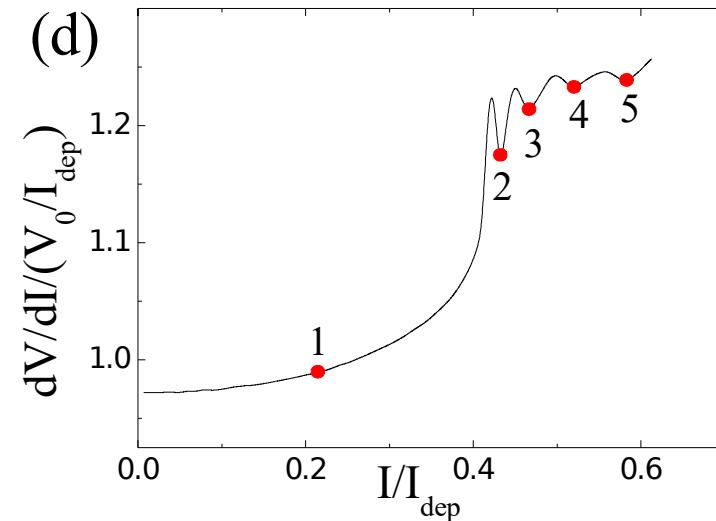
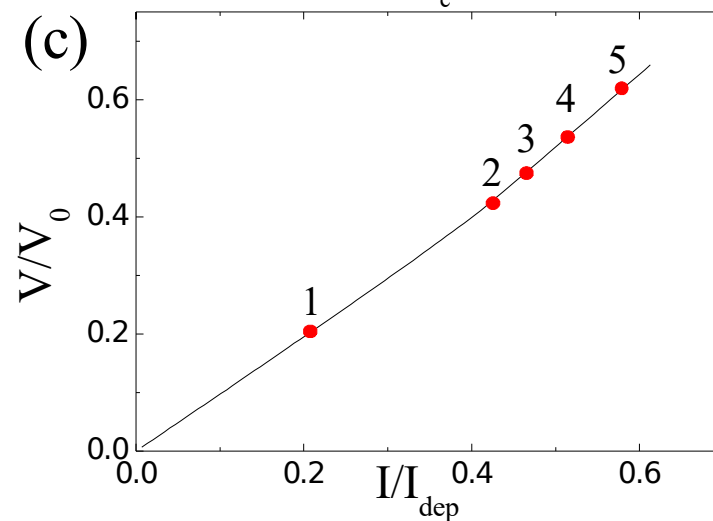
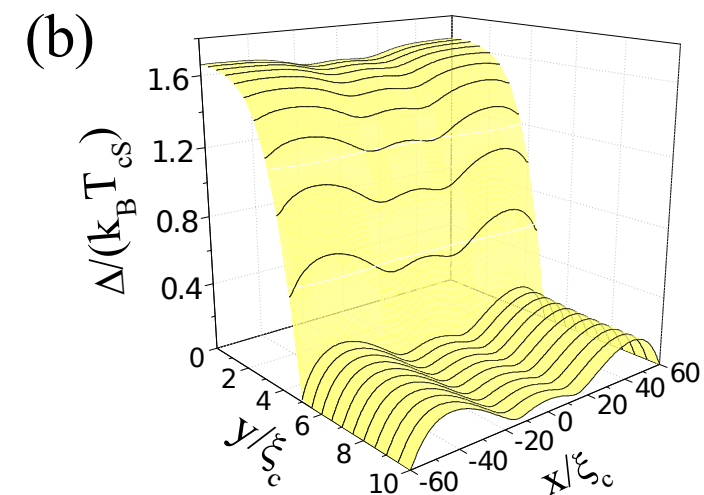
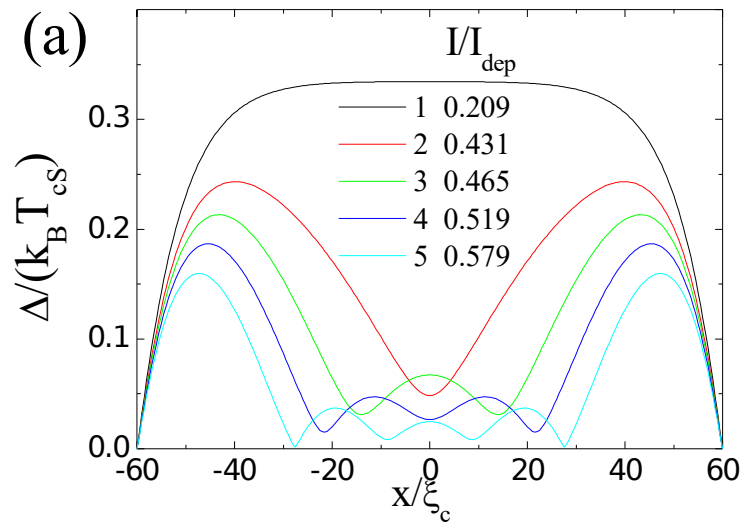


2D time-dependent Ginzburg-Landau model.

Electric field penetrates SN bridge on distance $L_E \sim (D/\Delta^2)^{1/2}$.

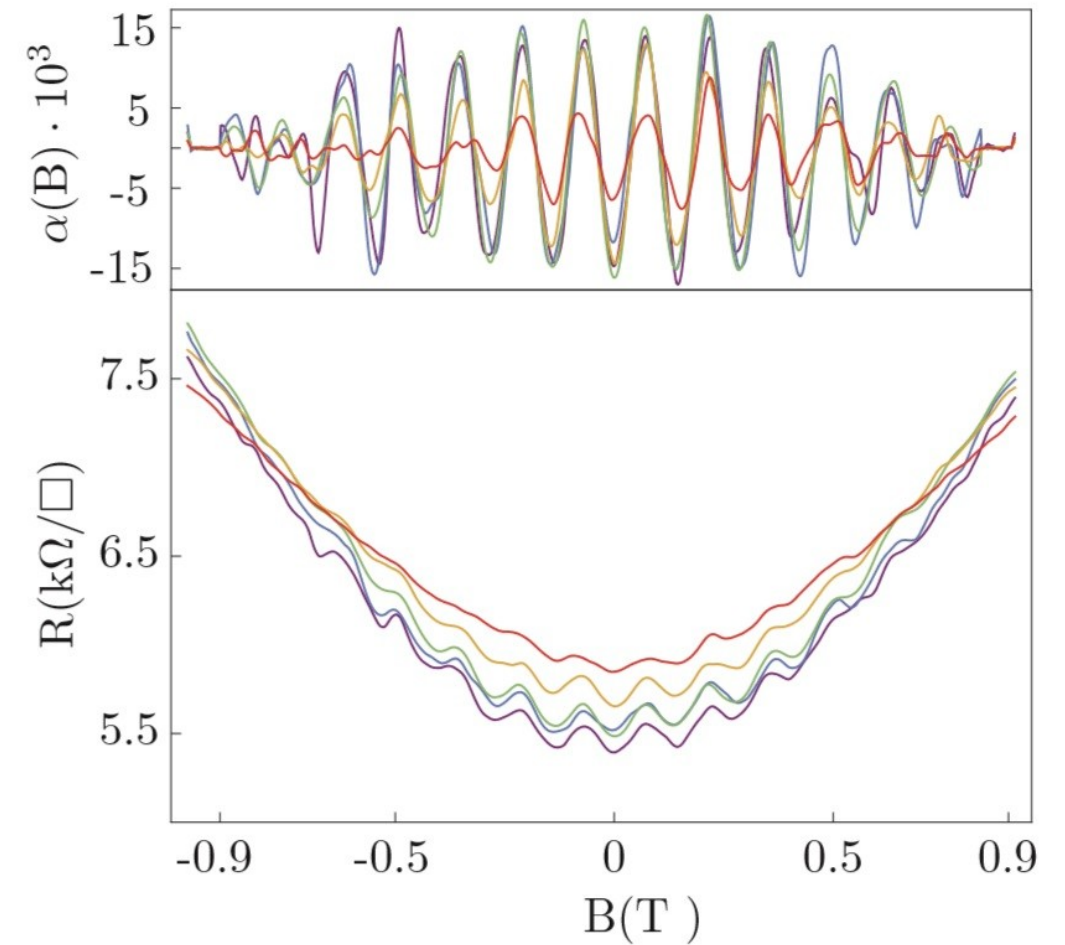
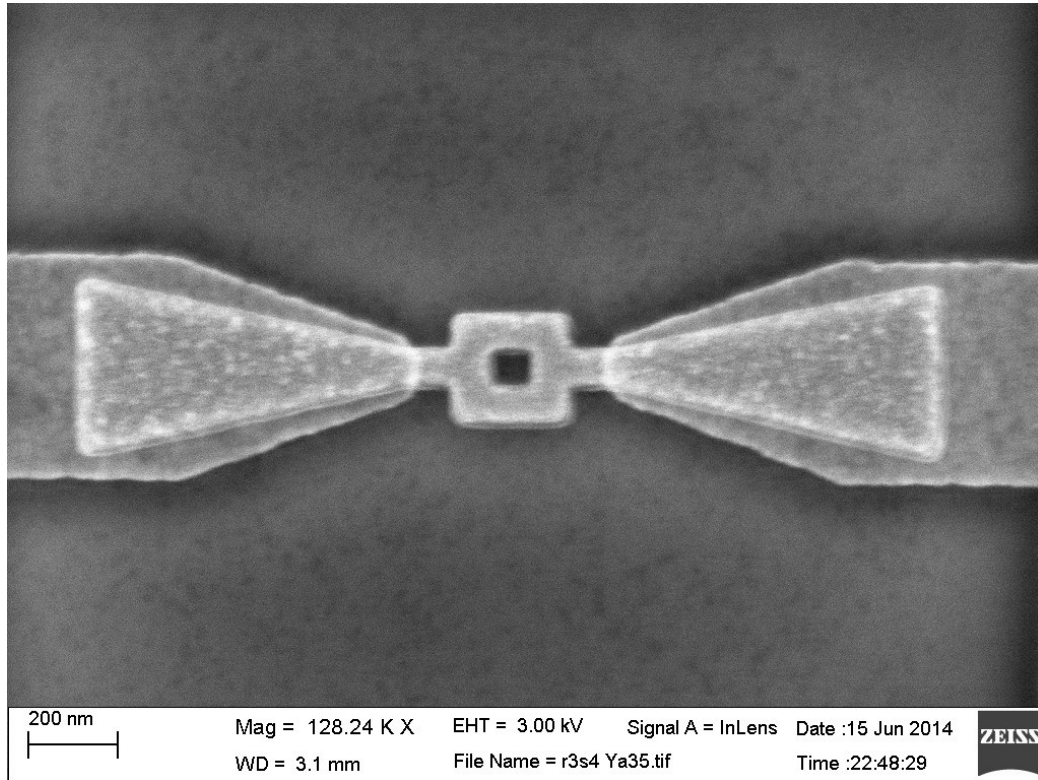
Nascent vortex has core with finite Δ and zero vorticity.

$$\xi_c = (\hbar D_S / k_B T_{c0})^{1/2} \ll \xi_N = (\hbar D_N / k_B T_{c0})^{1/2}$$



Little-Parks effect

Wikipedia



Little-Parks oscillations – proof of fluxoid (magnetic flux) quantization, vortices

Origin of name 'nascent vortex'

PRL 1974

PHYSICAL REVIEW

VOLUME 170, NUMBER 2

10 JUNE 1968

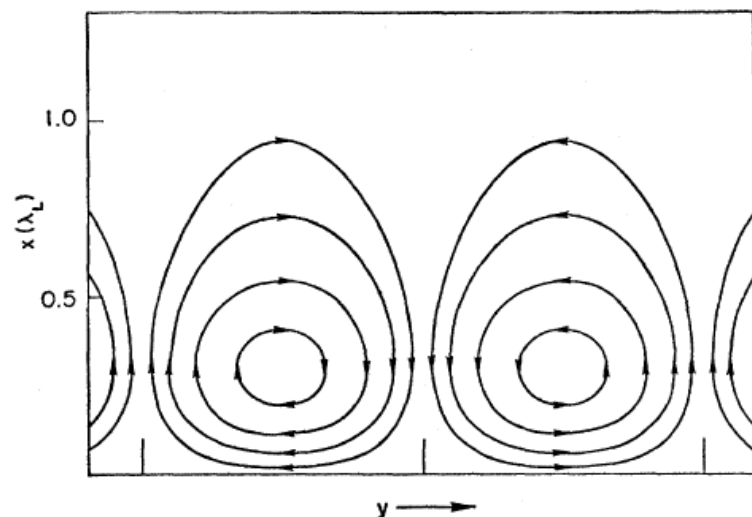
Stability Limits of the Meissner State and the Mechanism of Spontaneous Vortex Nucleation in Superconductors*

L. KRAMER†

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(Received 27 December 1967)

The limits of metastable existence of the superconducting Meissner state in a magnetic field are found by examining the second variation $\delta^2\Omega$ of the Ginzburg-Landau free energy. No assumptions about boundary conditions are made, and all possible fluctuations are examined. First, confining the fluctuations to one dimension, we show that $\delta^2\Omega$ is positive definite exactly up to that field H_{s2} (first calculated by Ginzburg) at which the Meissner state ceases to exist as a Ginzburg-Landau solution. At H_{s2} , the normal state penetrates spontaneously. Then we take into account arbitrary fluctuations and show that for superconductors with $\kappa \gtrsim 0.5$ another instability occurs at a lower field H_{s1} , leading to a new metastable modification of the Meissner state. **This new state possesses small vortices with fluxoid quantum zero along the boundary, and is metastable up to a field H_{s3} , which is probably of the order of H_{s2} ($H_{s3} = H_{s2} = H_c$ for $\kappa \gg 1$).** At H_{s3} , the normal state penetrates. Then, in a type-II superconductor with H_{s3} smaller than the upper critical field H_{c2} , spontaneous nucleation of Abrikosov vortices will take place in the normal region without violating fluxoid quantization. This should be the correct mechanism for vortex nucleation in ideal superheating experiments.



Nucleation of Vortices in the Superconducting Mixed State: Nascent Vortices*

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Evidence is presented that the mixed-state order parameter at surfaces parallel to the magnetic field is strongly modulated. The minima of this modulation act as the nucleation and denucleation sites for vortices.

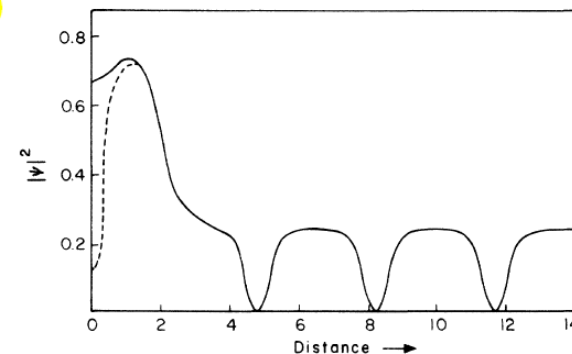


FIG. 1. The order parameter $|\psi|^2$ as a function of distance into the superconductor in units of a coherence length for $H = 0.6H_{c2}$ and $\kappa = 4$. The dashed line indicates the order-parameter minima (nascent vortices) at the surface.

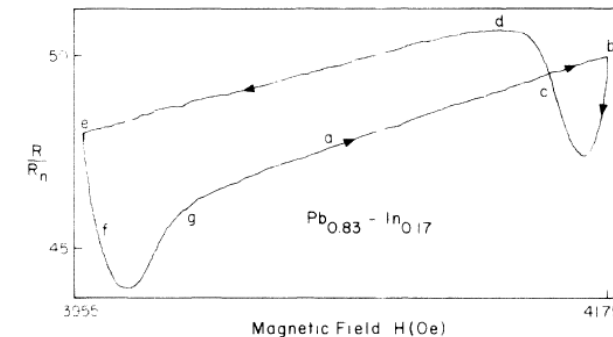
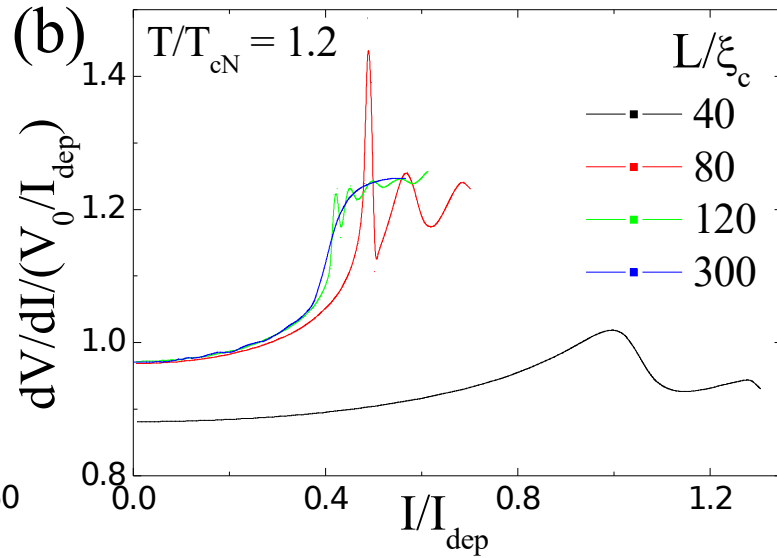
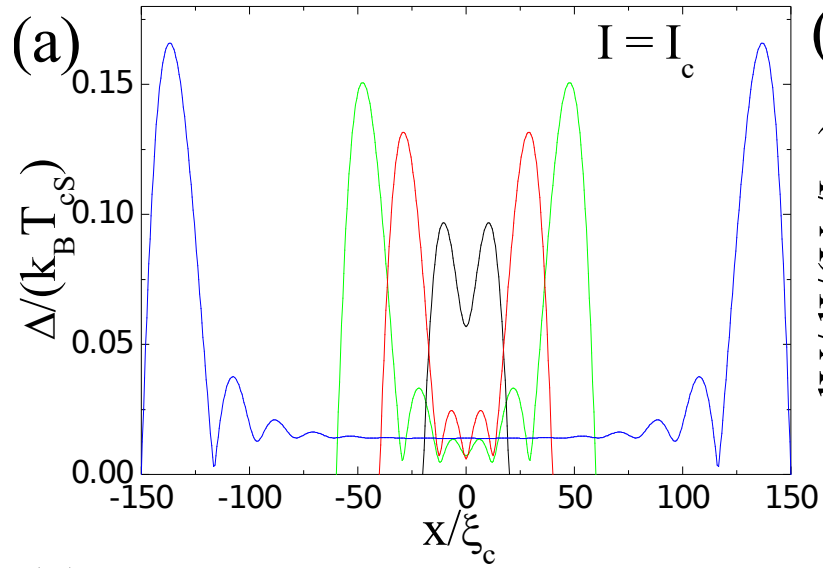
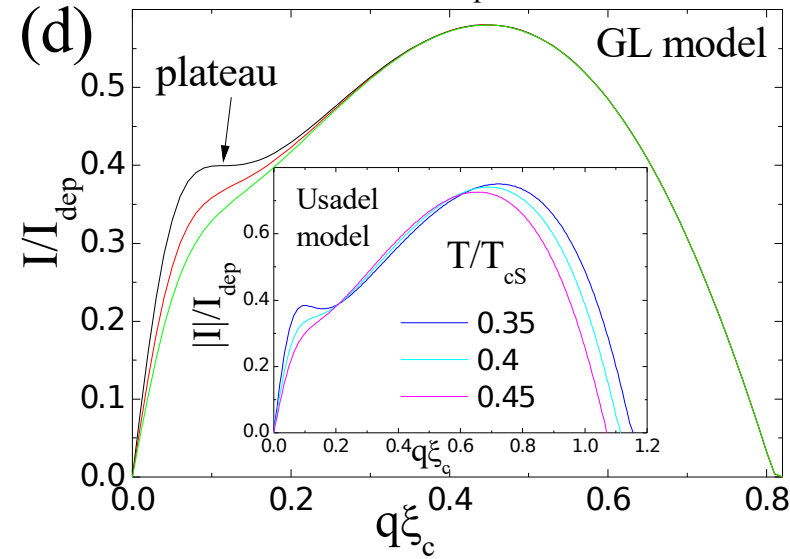
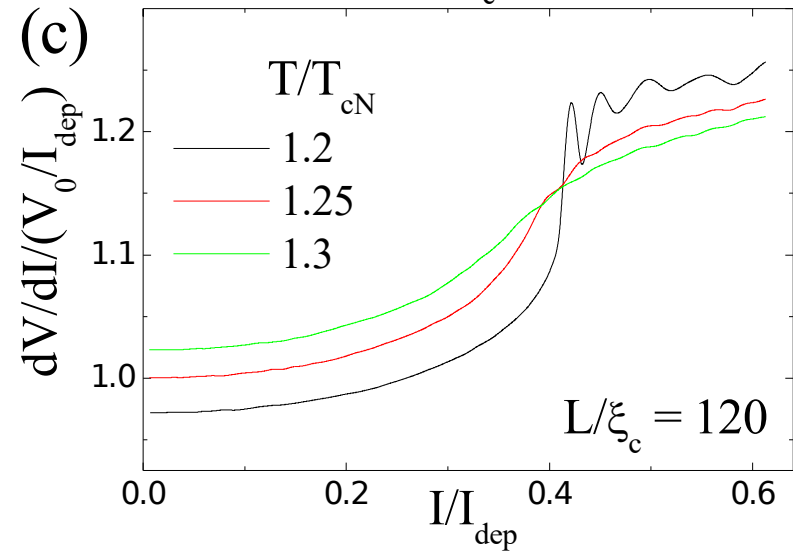


FIG. 2. A recorder tracing of the hysteresis loop of the microwave absorption ($f = 35$ GHz) at $H \approx 0.8H_{c2}$ and $T = 1.7^\circ\text{K}$. The magnetic field is parallel to the surface and perpendicular to the microwave currents.

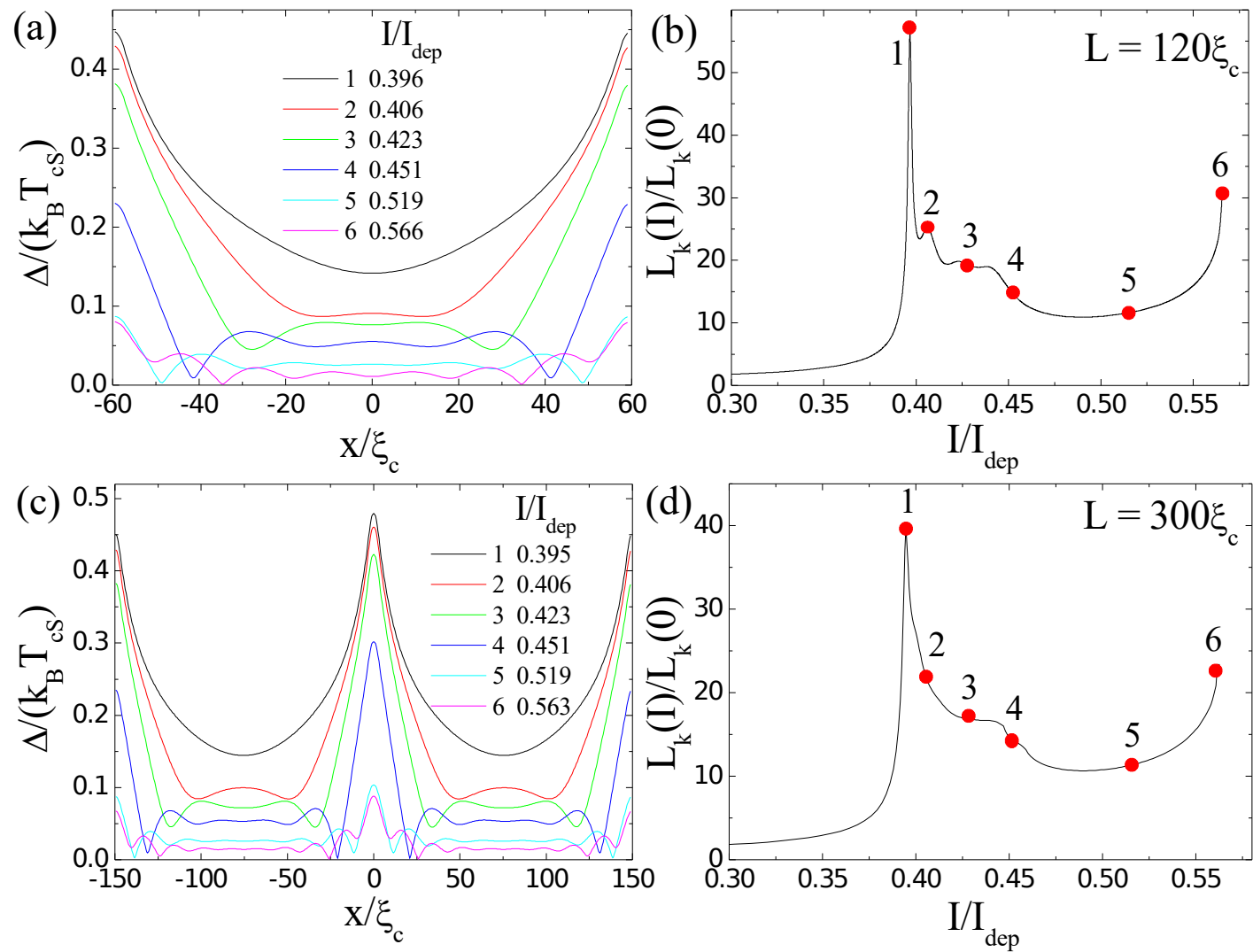
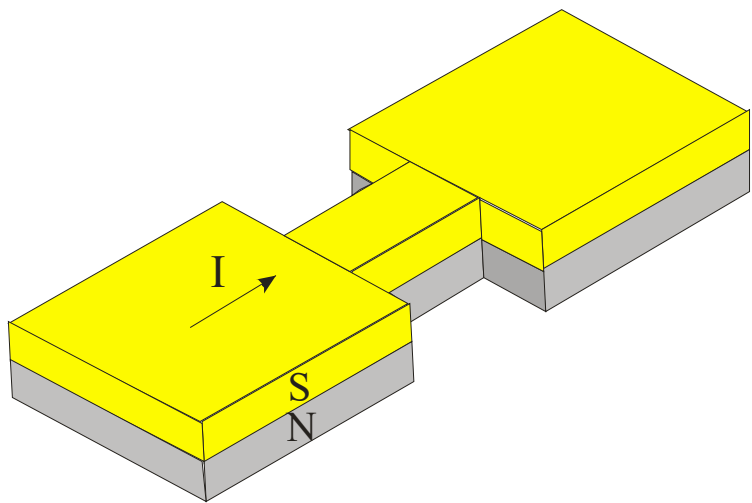
Nascent vortices in SN bridge



Nascent vortices exist when $D_N/D_S = \rho_S/\rho_N \gg 10$ and are well ‘visible’ when plateau on $I(q)$ appears for short enough bridge ($\xi_c \sim 6-7$ nm).



Nascent vortices in SN bridge with superconducting leads



Conclusion

SN hybrid composed of dirty thin superconducting and clean normal metal layers has peculiar superconducting properties. Namely:

1. N-layer may provide the dominant contribution to the diamagnetic response of whole bilayer structure at low temperatures.
2. The presence of N-layer may considerably increase the critical current I_c .
3. SN hybrid placed in in-plane magnetic field has finite momentum even when it is in state with $I=0$. In finite momentum state $L_k(I)$ is nonreciprocal and there is giant superconducting diode effect (origin of its large value and ‘wrong’ sign is not clear at large B_{in}).
4. SN bridge/strip has huge nonlinearity of kinetic inductance at current less than depairing current. This property could be used in different applications (single photon detectors, magnetometers, parametric amplifiers).
5. There is a prediction that SN bridge may host nascent vortices (sub- or pre-vortices with zero vorticity), which can be seen via oscillations of differential resistance and/or kinetic inductance of SN bridge with change of the current.

Equations

Usadel equation:

$$\hbar D \frac{\partial^2 \Theta}{\partial x^2} - \left(2\hbar\omega_n + \frac{D}{\hbar} q^2 \cos\Theta \right) \sin\Theta + 2\Delta \cos\Theta = 0$$

Self-consistency equation:

$$\Delta \ln \left(\frac{T}{T_{c0}} \right) + 2\pi k_B T \sum_{\omega_n \geq 0} \left(\frac{\Delta}{\omega_n} - \text{Re} \sin\Theta_s \right) = 0 \quad \hbar\omega_n = \pi k_B T (2n + 1)$$

Superconducting current density:

$$j = \frac{\sigma_n 2\pi k_B T}{e\hbar} q_s \sum_{\omega_n \geq 0} \text{Re}(\sin^2\Theta)$$

Inverse London penetration depth:

$$\frac{1}{\lambda^2} = \frac{16\pi^2 \sigma_n k_B T}{\hbar c^2} \sum_{\omega_n \geq 0} \text{Re}(\sin^2\Theta)$$